



# Excited-state non-adiabatic quantum dynamics and its theoretical challenges

Zernike Institute for Advanced Materials

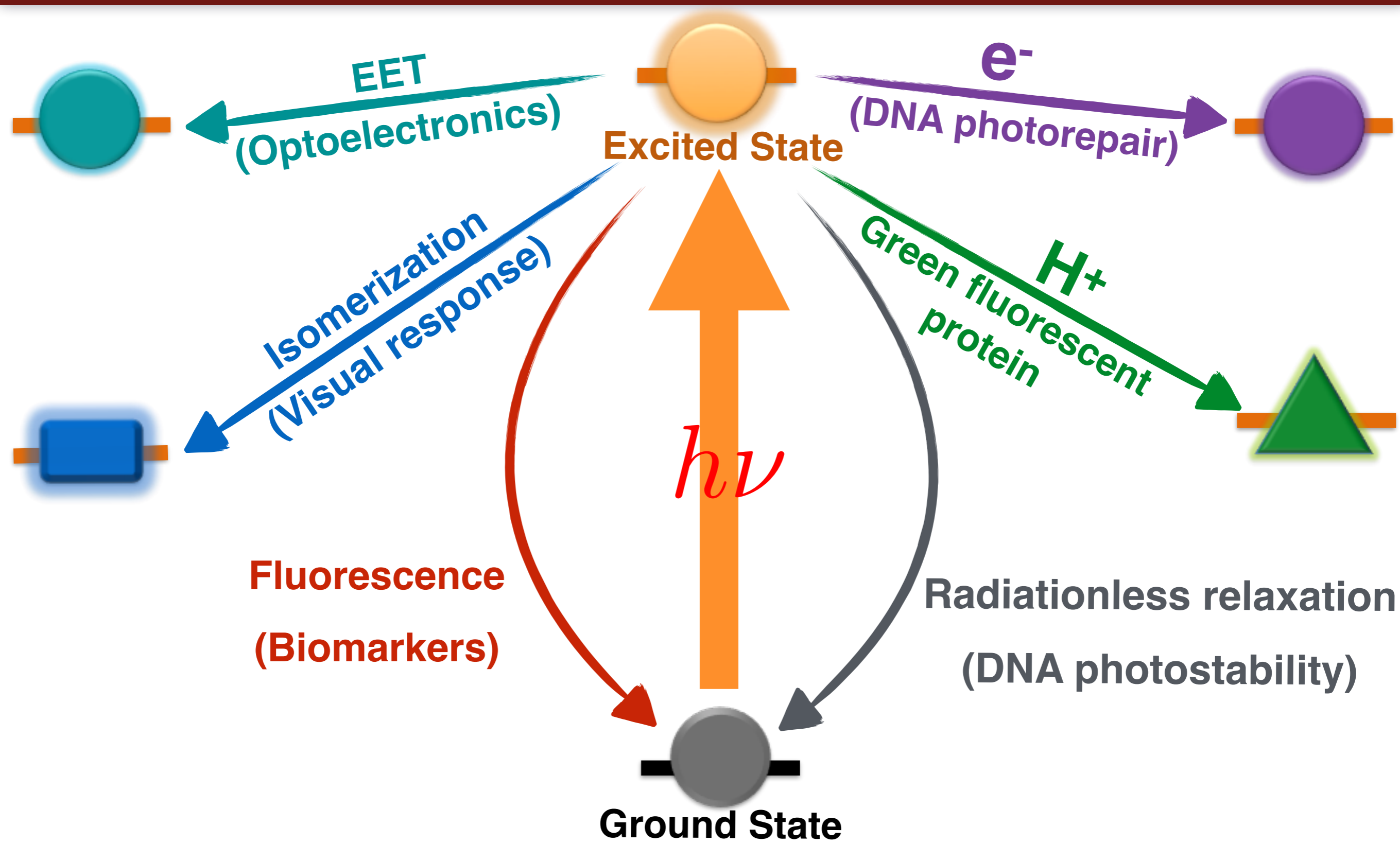
University of Groningen

Shirin Faraji

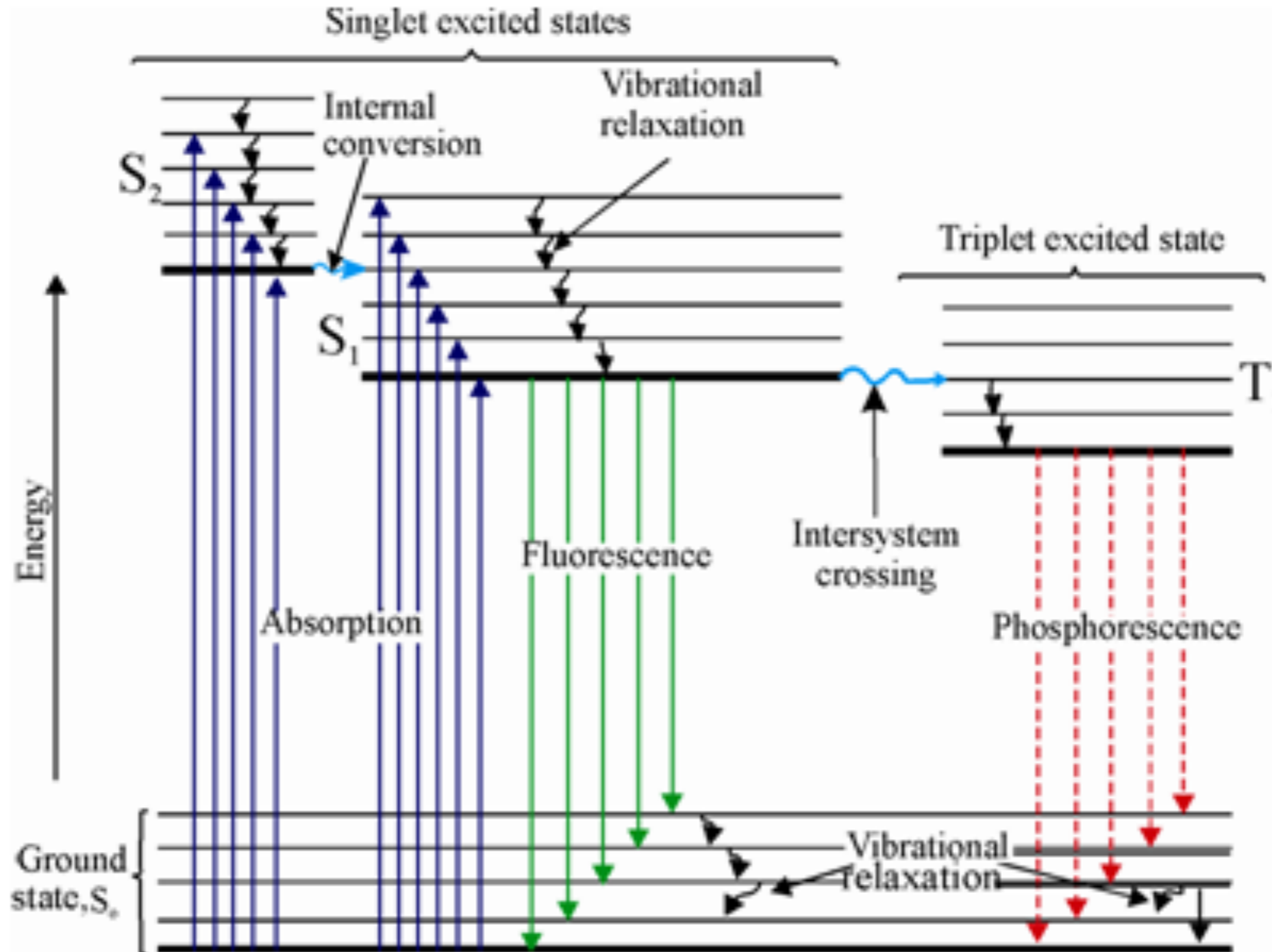
[s.s.faraji@rug.nl](mailto:s.s.faraji@rug.nl)



# Photochemical processes in complex environment

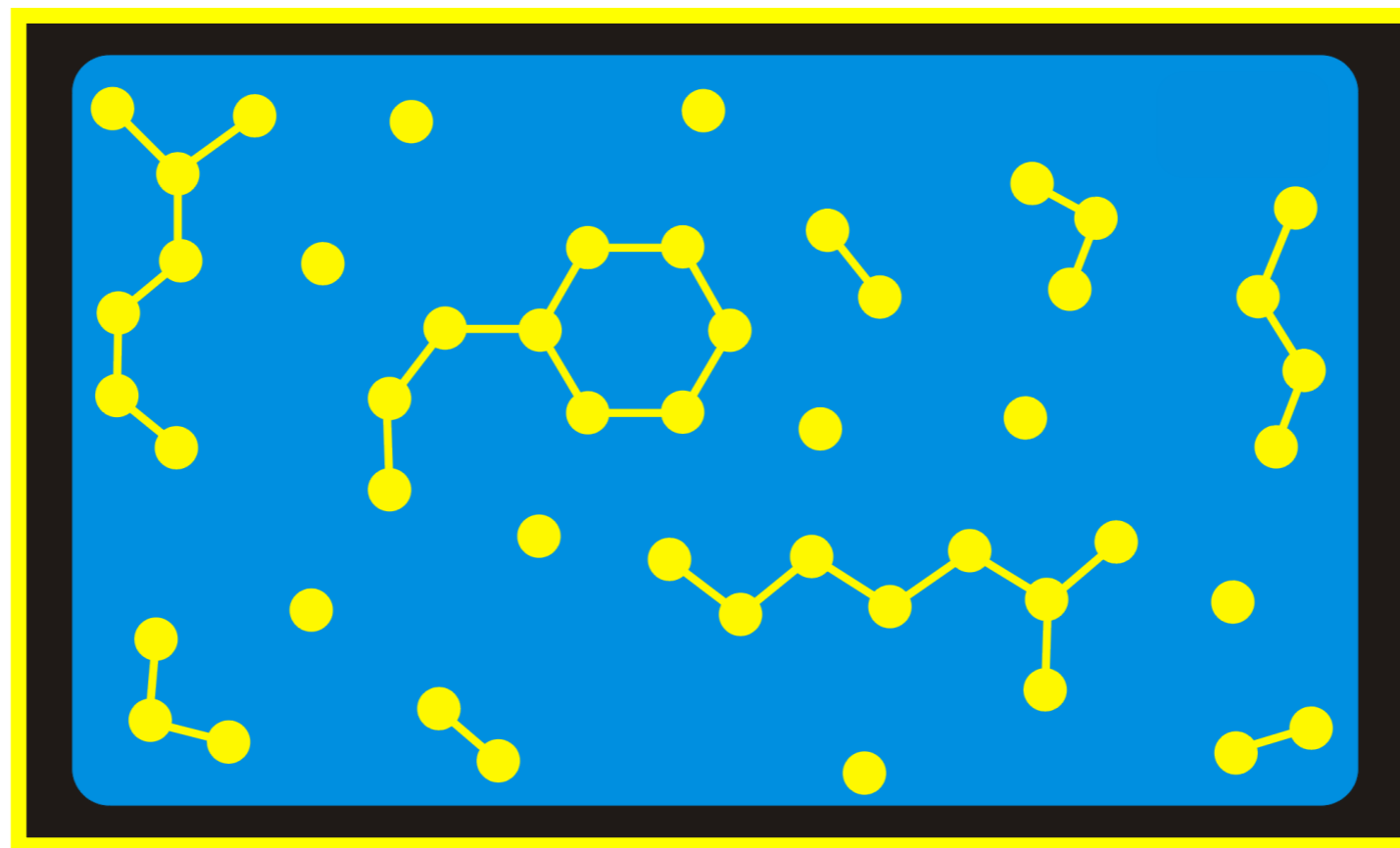


# Jablonski diagram

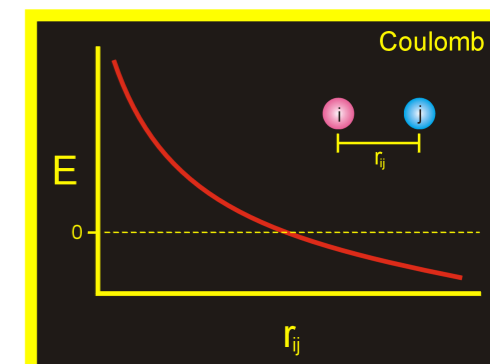
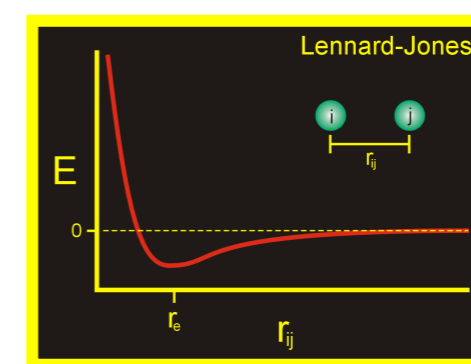
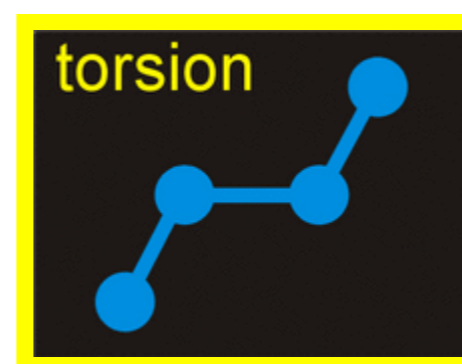
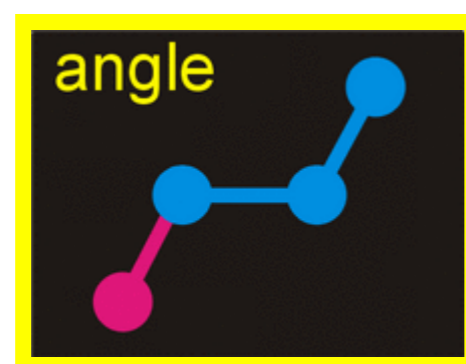
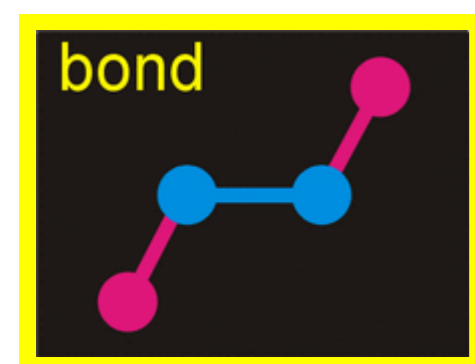


# Molecular mechanics

## Newtonian equation of motion



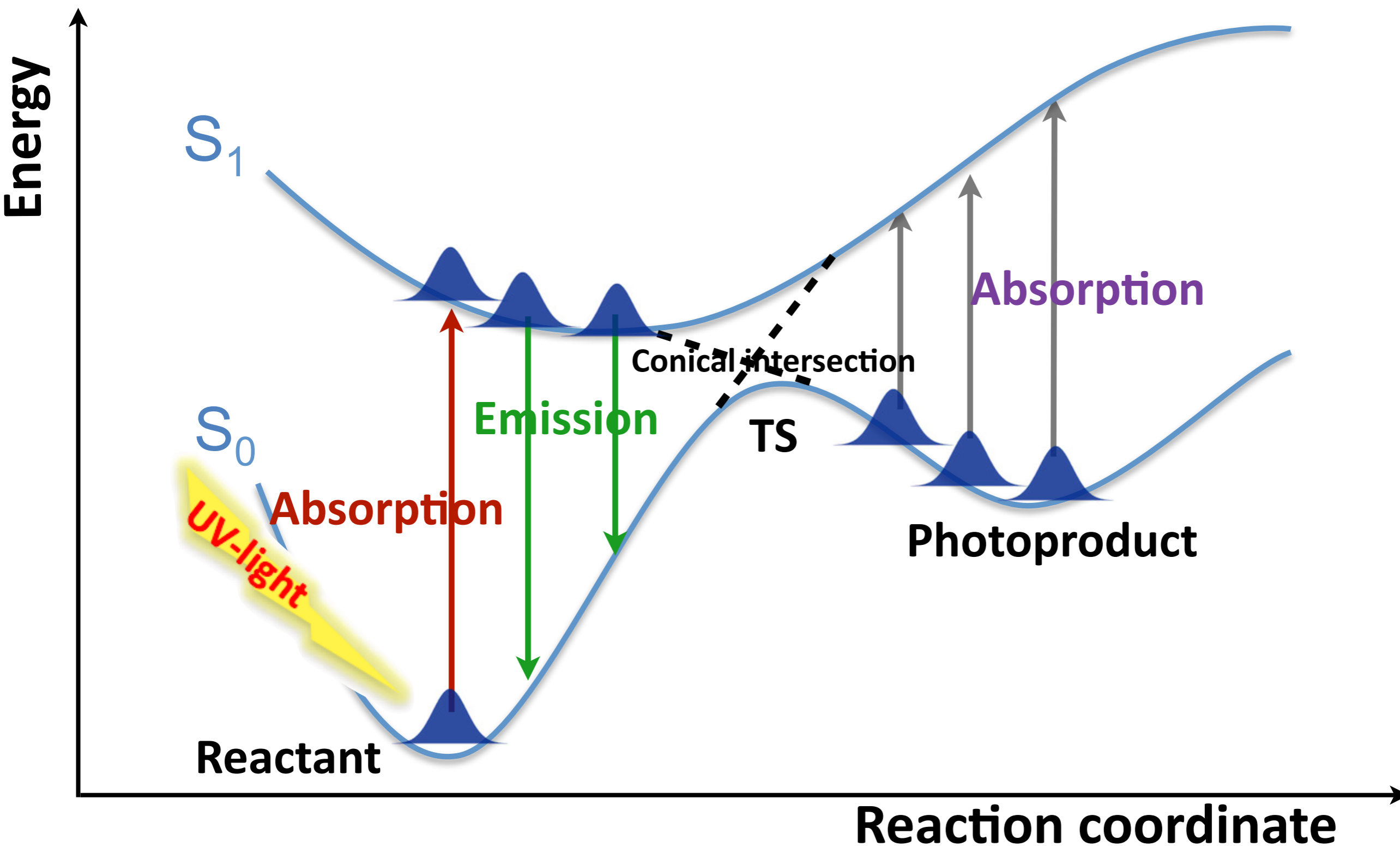
## Classical force fields



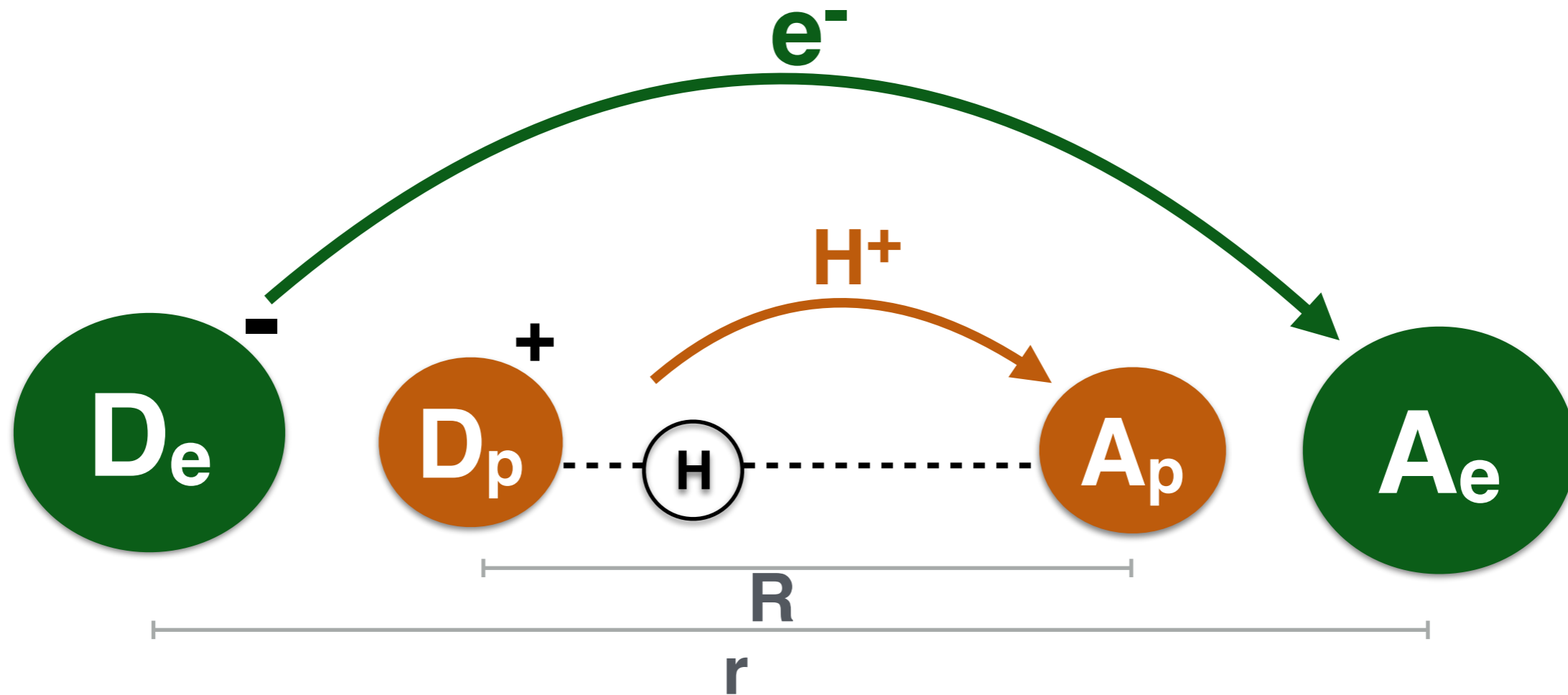
$$v_b(r) = \frac{1}{2}k_b(r - r_0)^2 \quad v_a(\theta) = \frac{1}{2}k_a(\theta - \theta_0)^2 \quad v_d(\varphi) = k_d(1 + \cos(n\varphi - \varphi_0)) \quad v_{ij}(r_{ij}) = \frac{C_{ij}^{(12)}}{r_{ij}^{12}} - \frac{C_{ij}^{(6)}}{r_{ij}^6} \quad v_c(r_{ij}) = \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$



# Theoretical challenges in quantum dynamics



# Proton Coupled Electron Transfer (PCET)



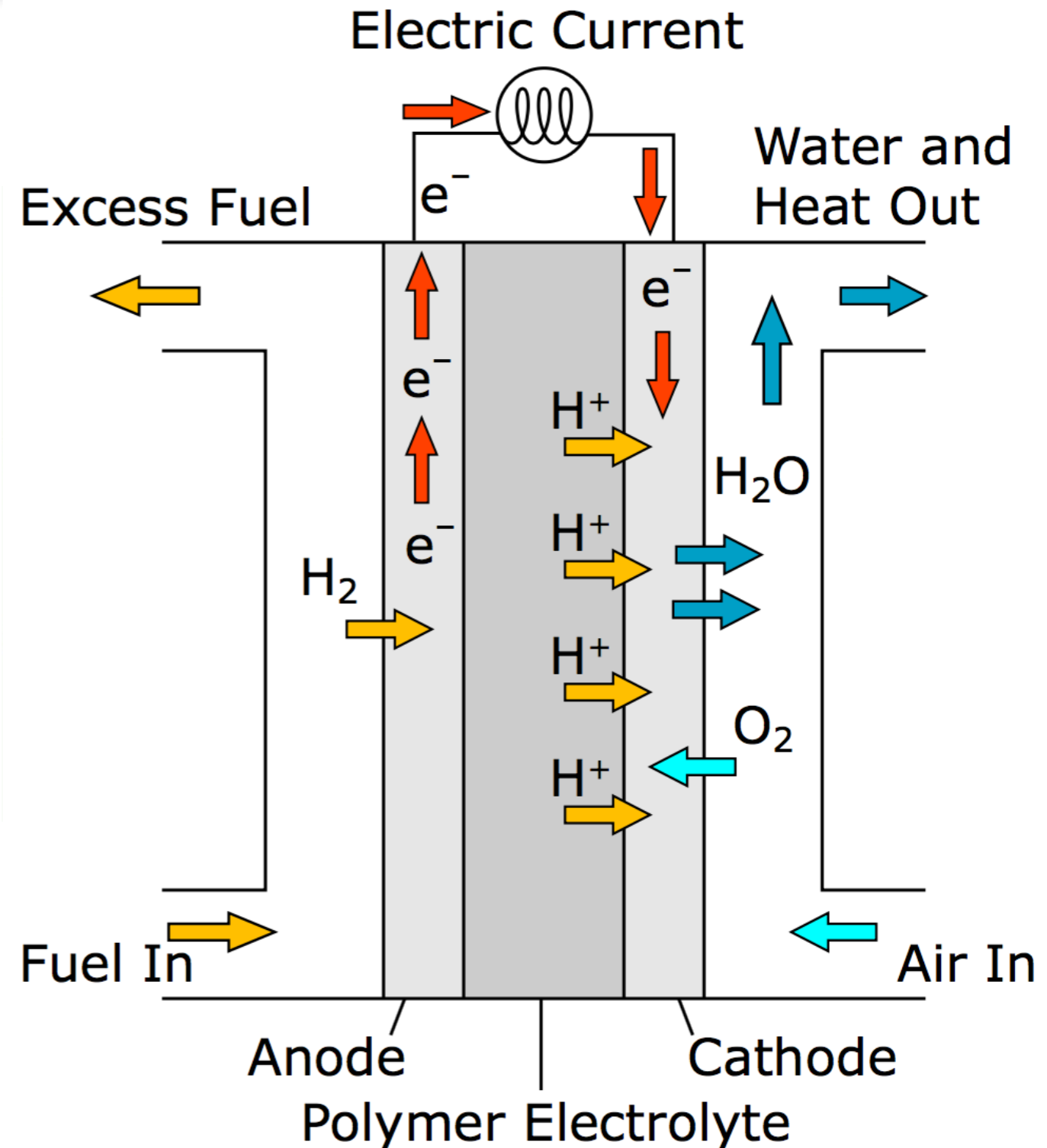
# Importance of PCET

## Biological processes

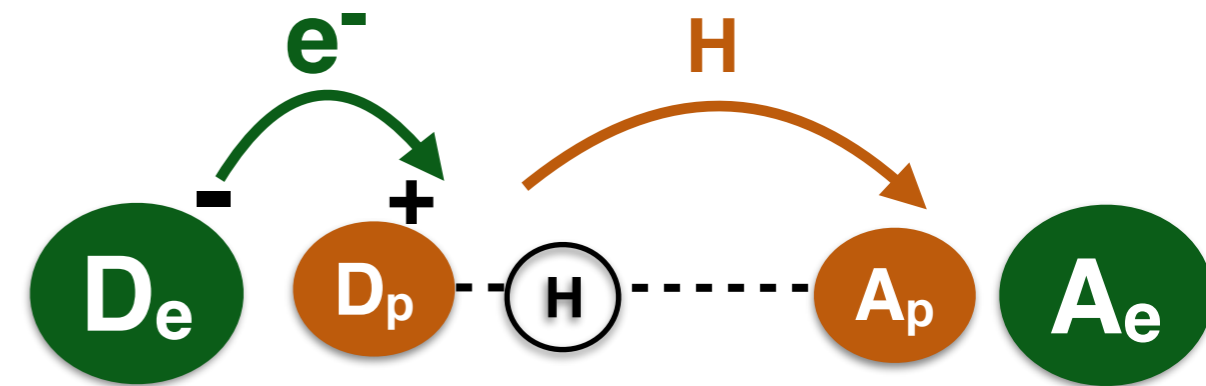
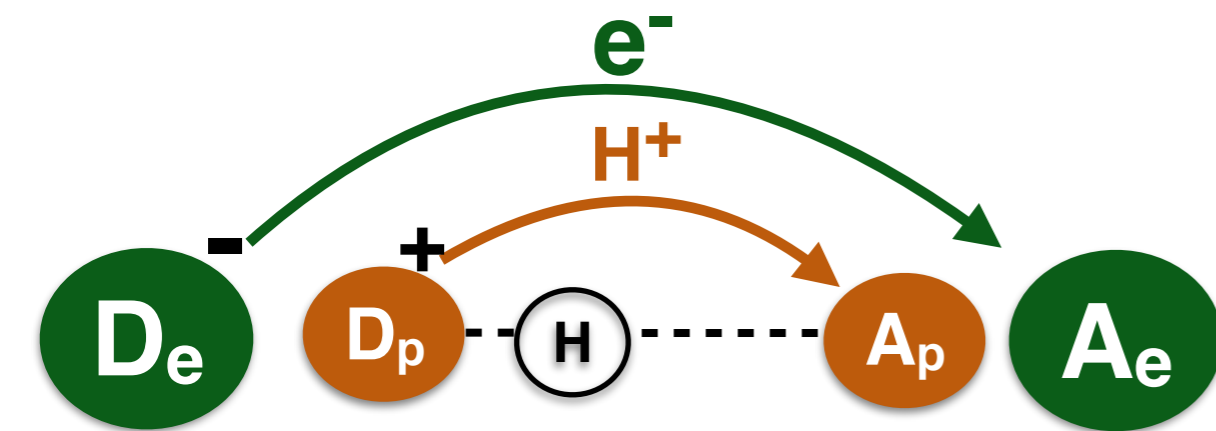
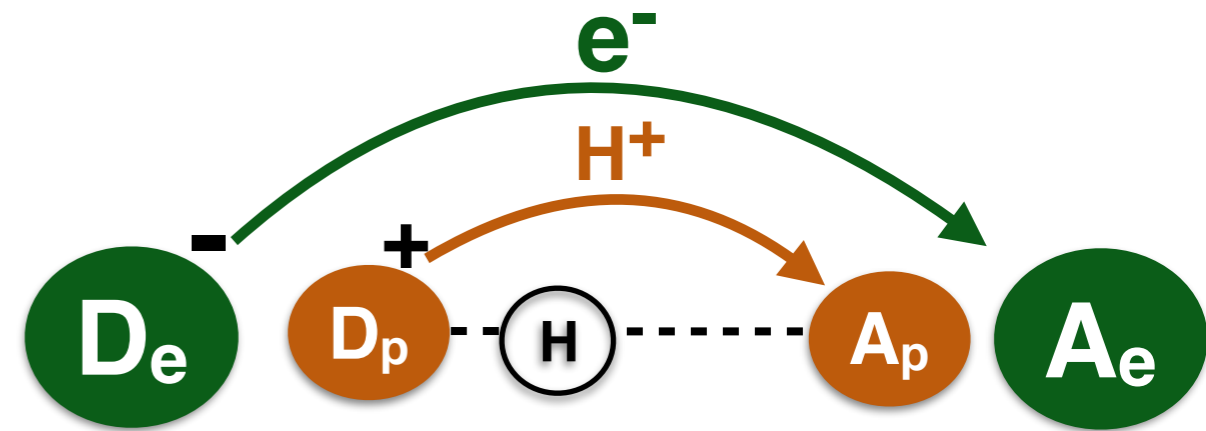
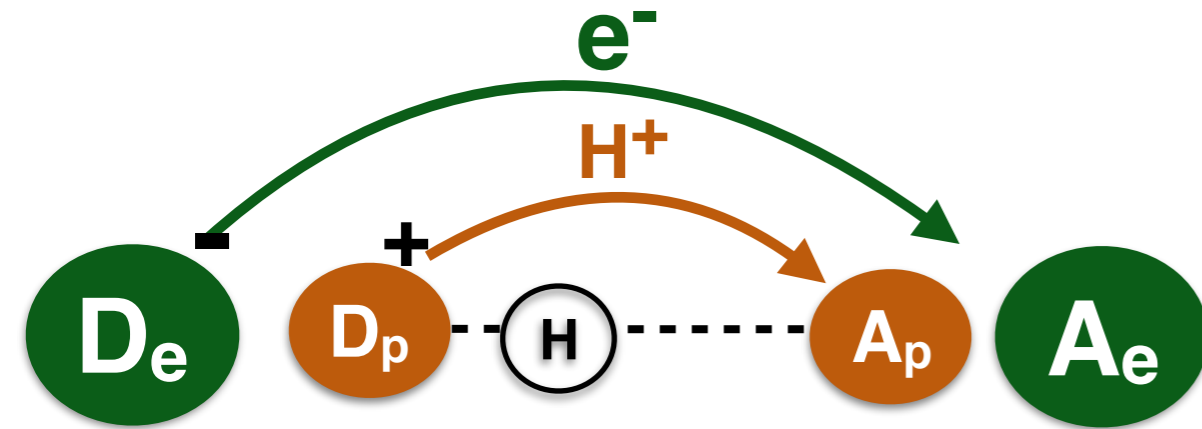
- Photosynthesis
- Respiration
- Enzymatic reactions
- Light-induced DNA repair

## Electrochemical processes

- Fuel cells
- Solar cells
- Energy devices

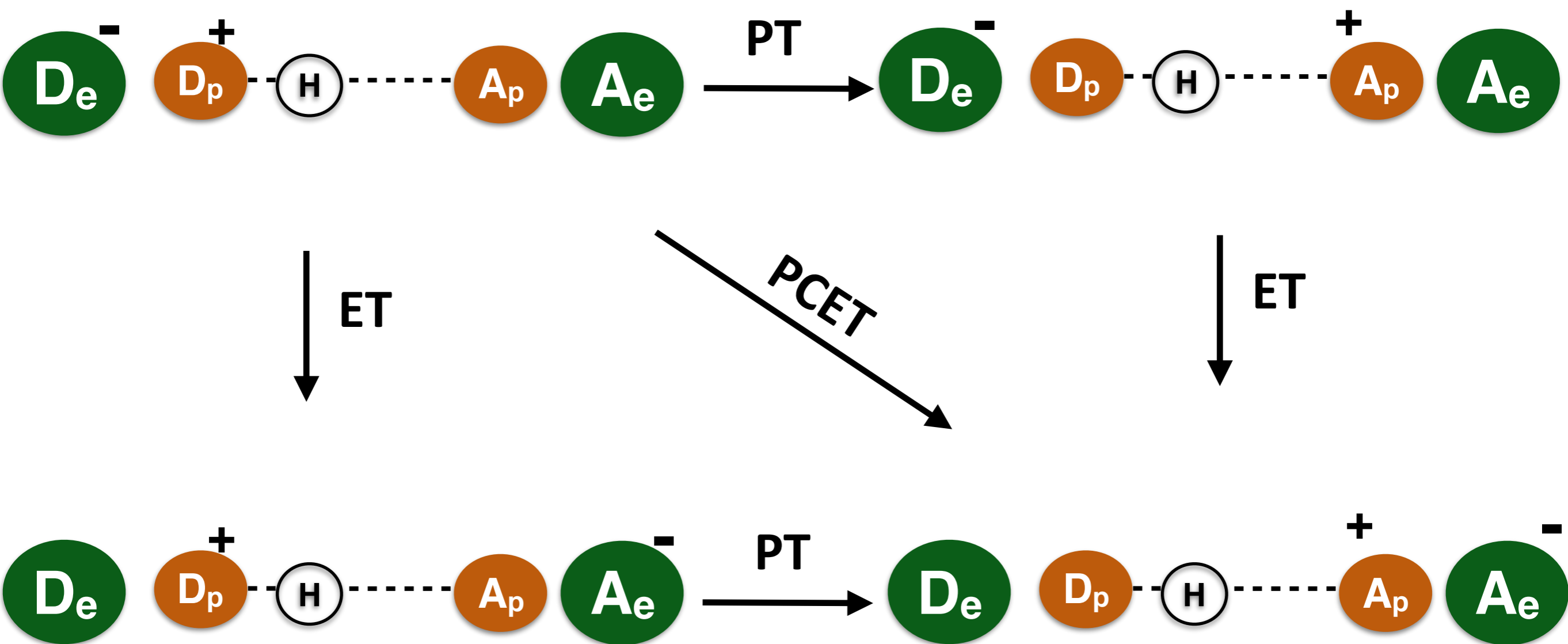


# Sequential vs. Concerted



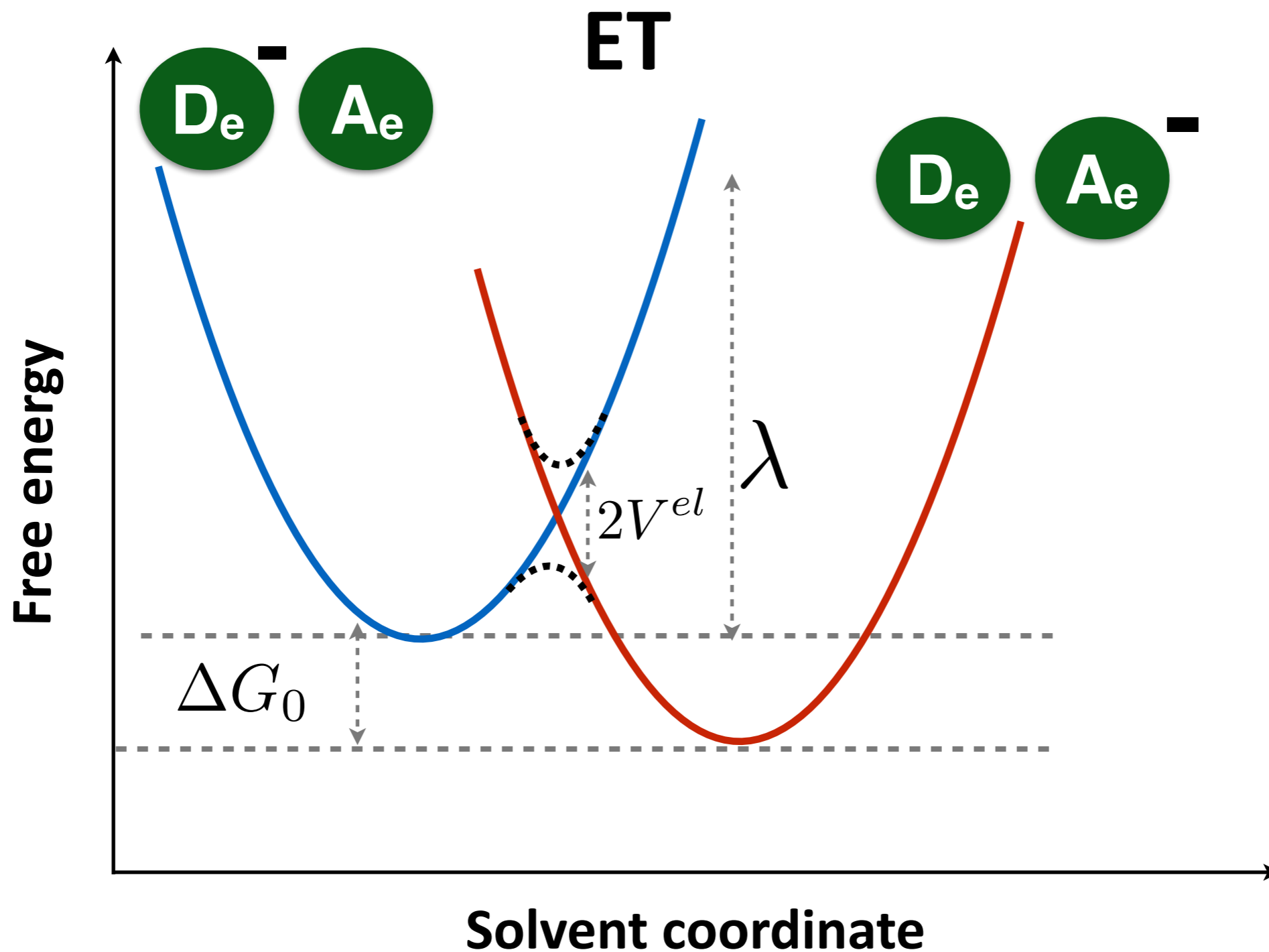


# 4 States



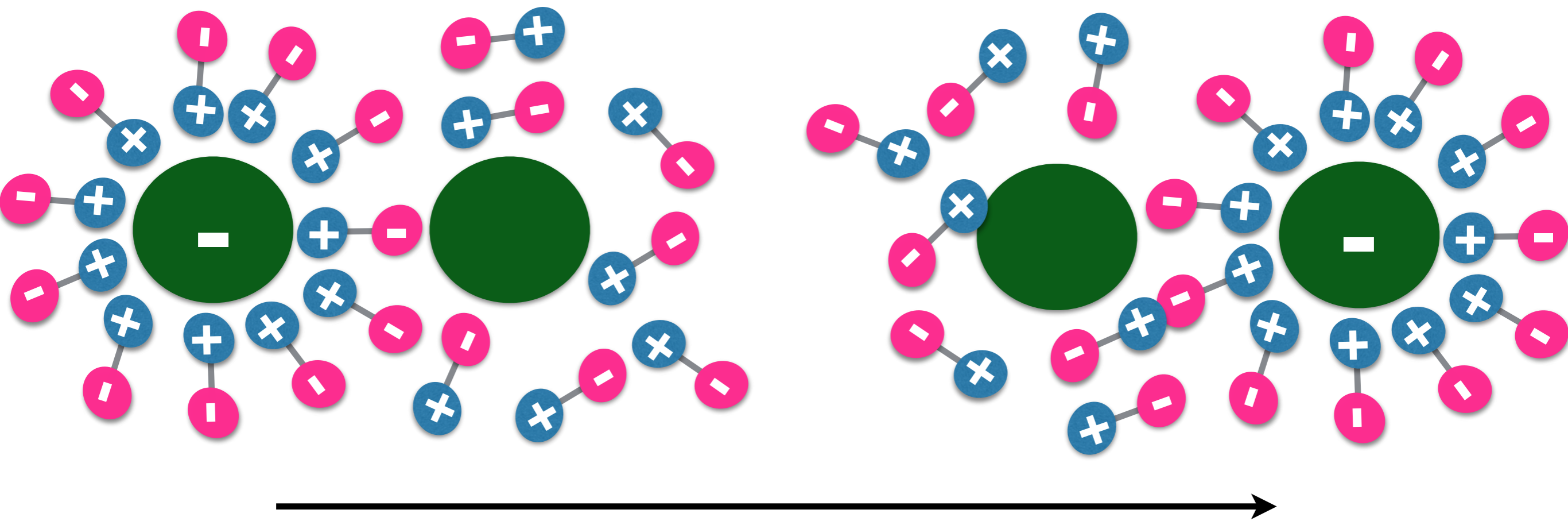
**PCET avoids high energy charged intermediates**

# Single electron transfer (ET)



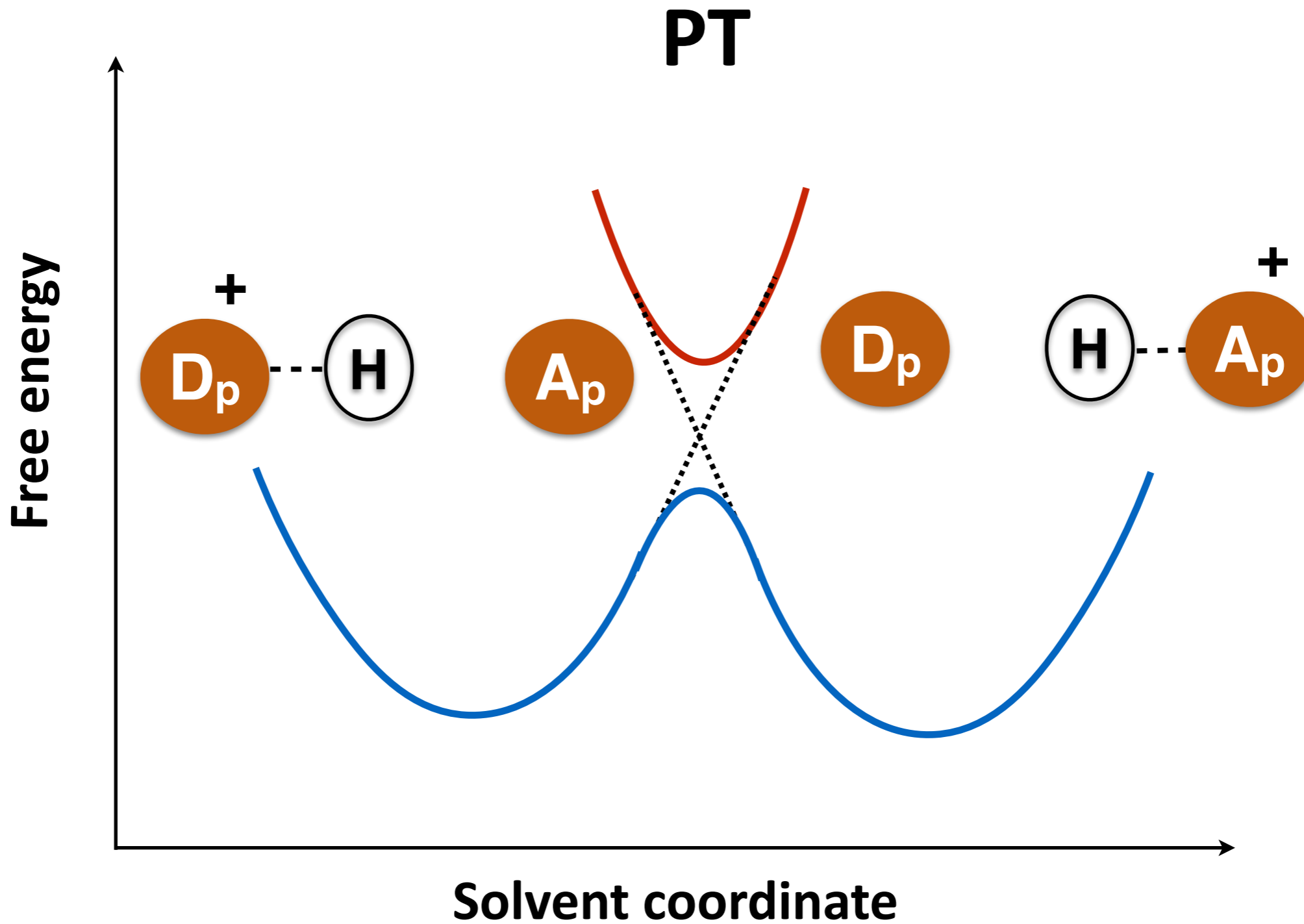
R. A. Marcus, Nobel Lecture, *Angew. Chem. Int. Ed. Engl.*, 32: 1111, 1993.

# Environment coordinate



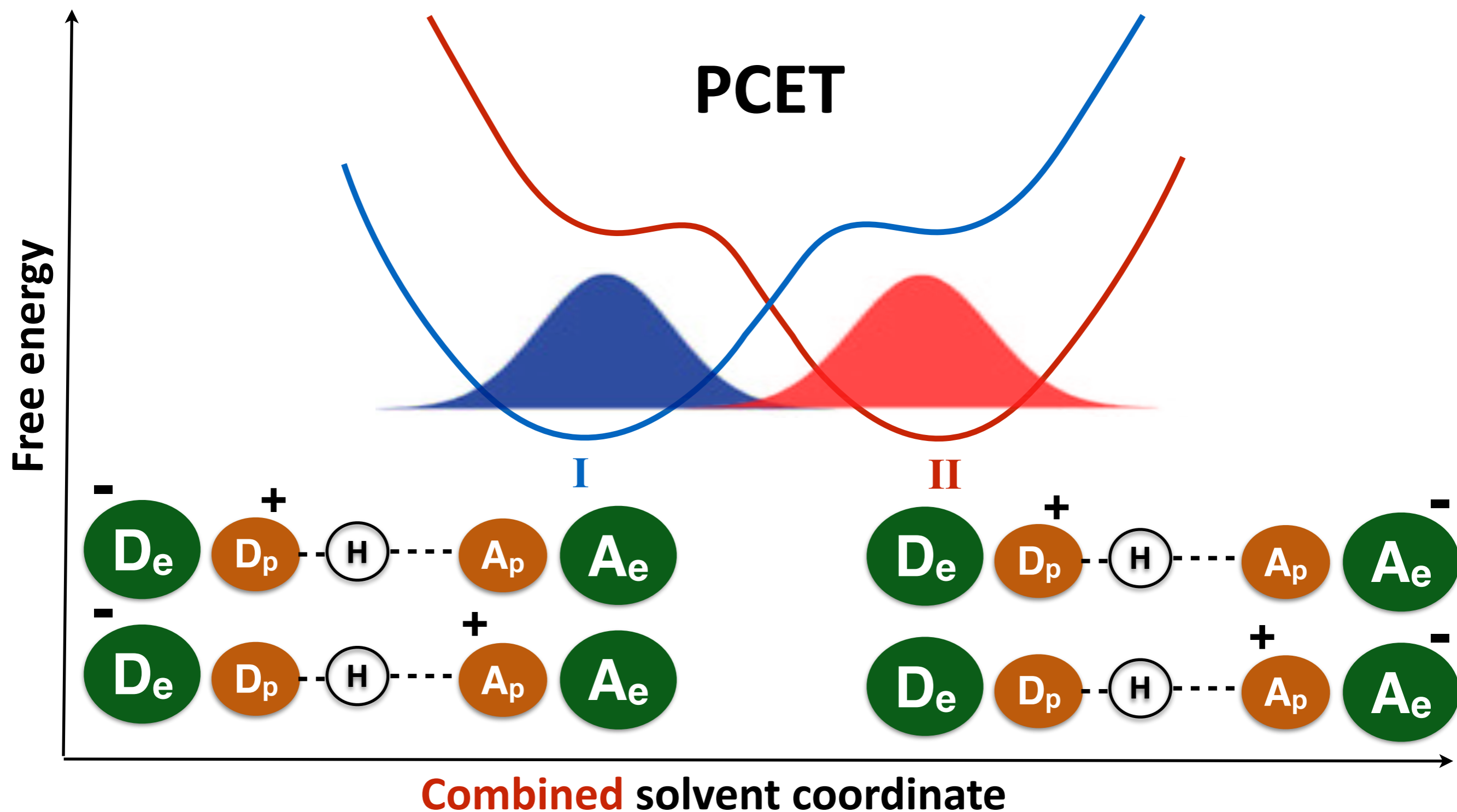
Solvent coordinate

# Single proton transfer (PT)



D. Borgis, J.T. Hynes, *Chem. Phys.*, 170, 315, 1993.

# Electron-proton vibronic states



S. Hammes-Schiffer, *J. Am. Chem. Soc.* 137, 8860-8871, 2015.

# Theoretical Challenges of PCET

## Wide range of timescales

- Electrons
- Transferring protons
- Solute modes
- Environment electronic/nuclear polarization

fs



ps

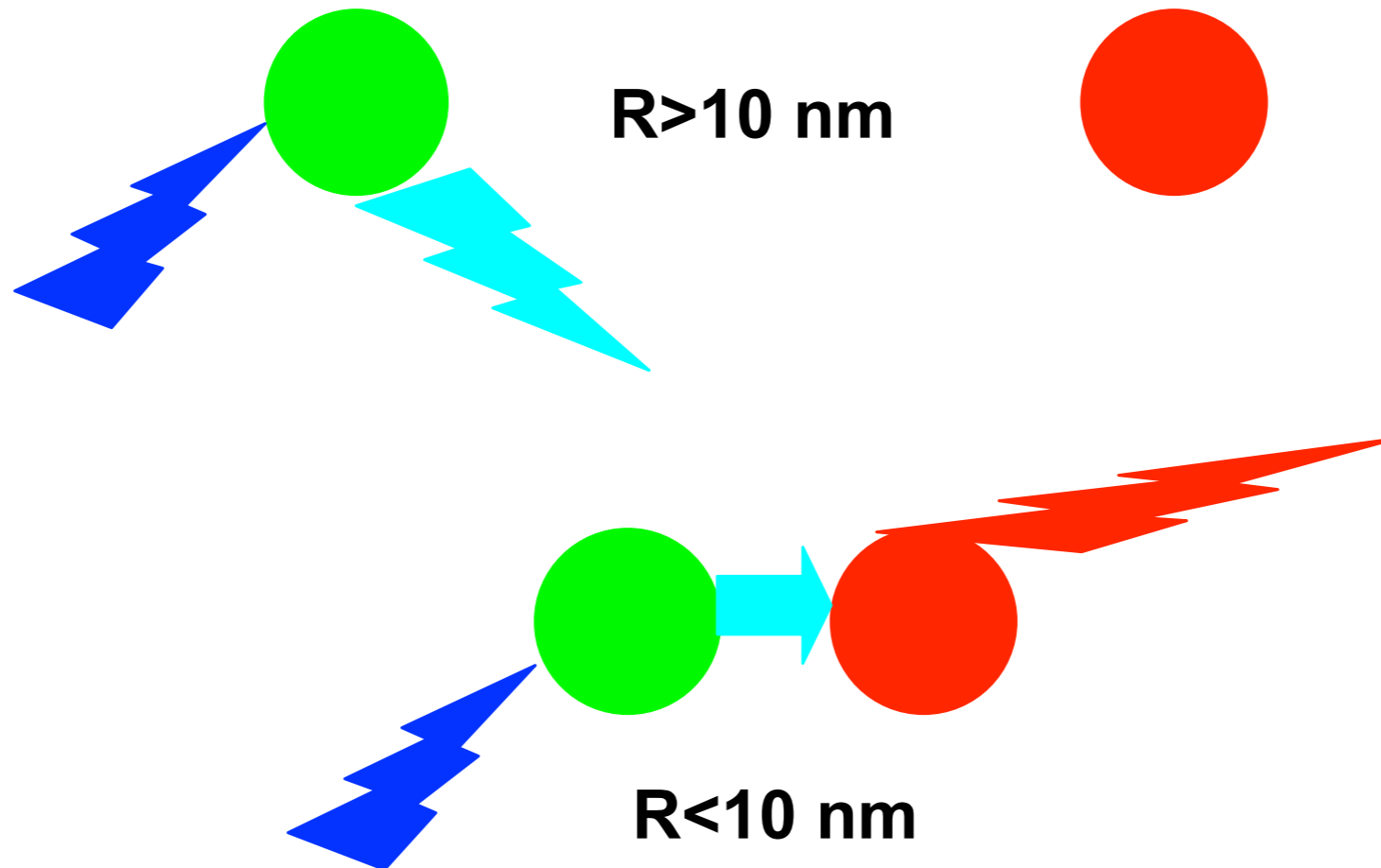
## Quantum behavior of electrons and protons

- Hydrogen/proton tunneling
- Excited electronic/vibrational states
- Mixed electron-nuclear motion (non-adiabatic effect)

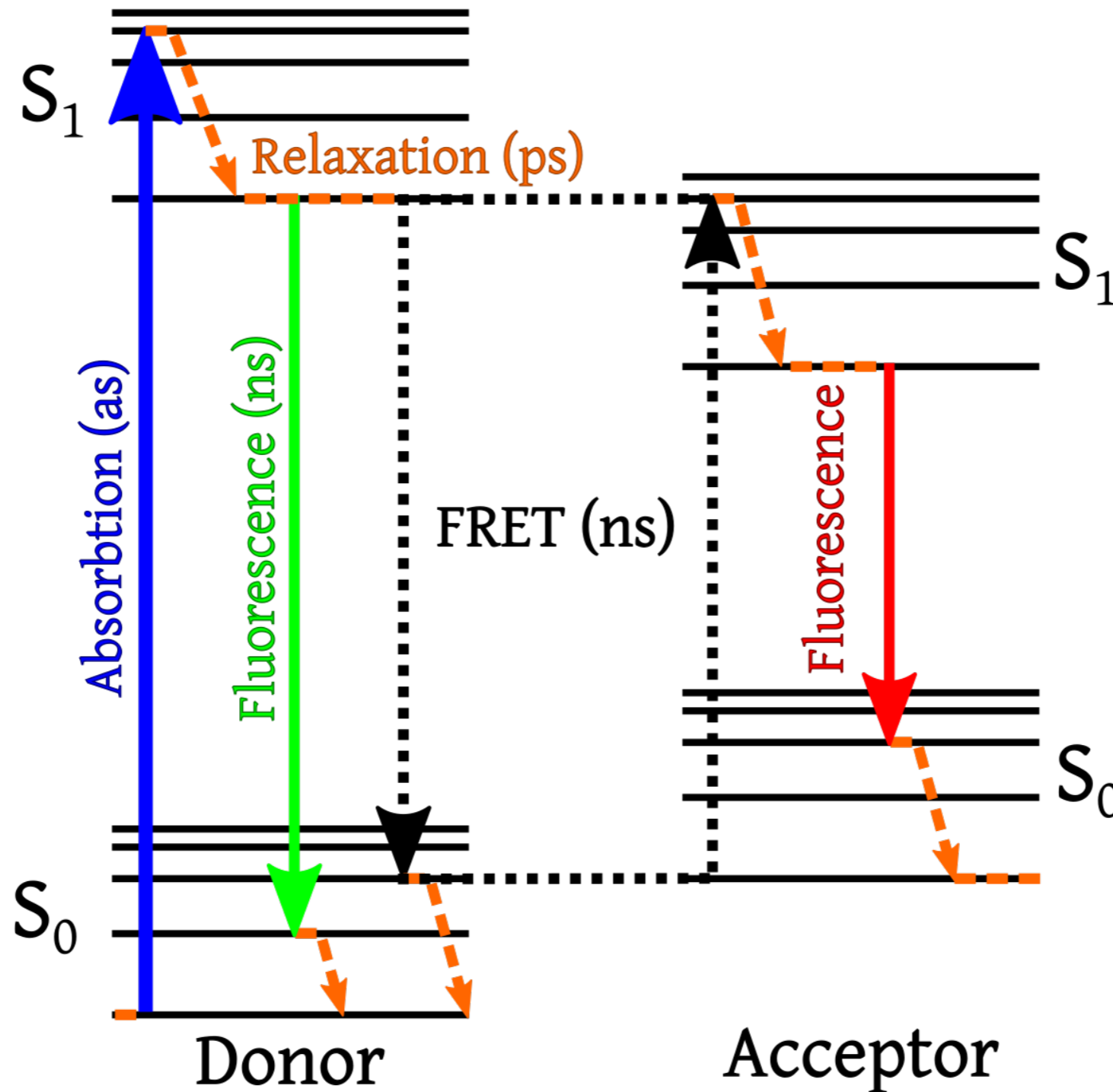
## Role of the Environment

- Size of the system

# Förster resonance energy transfer (FRET)

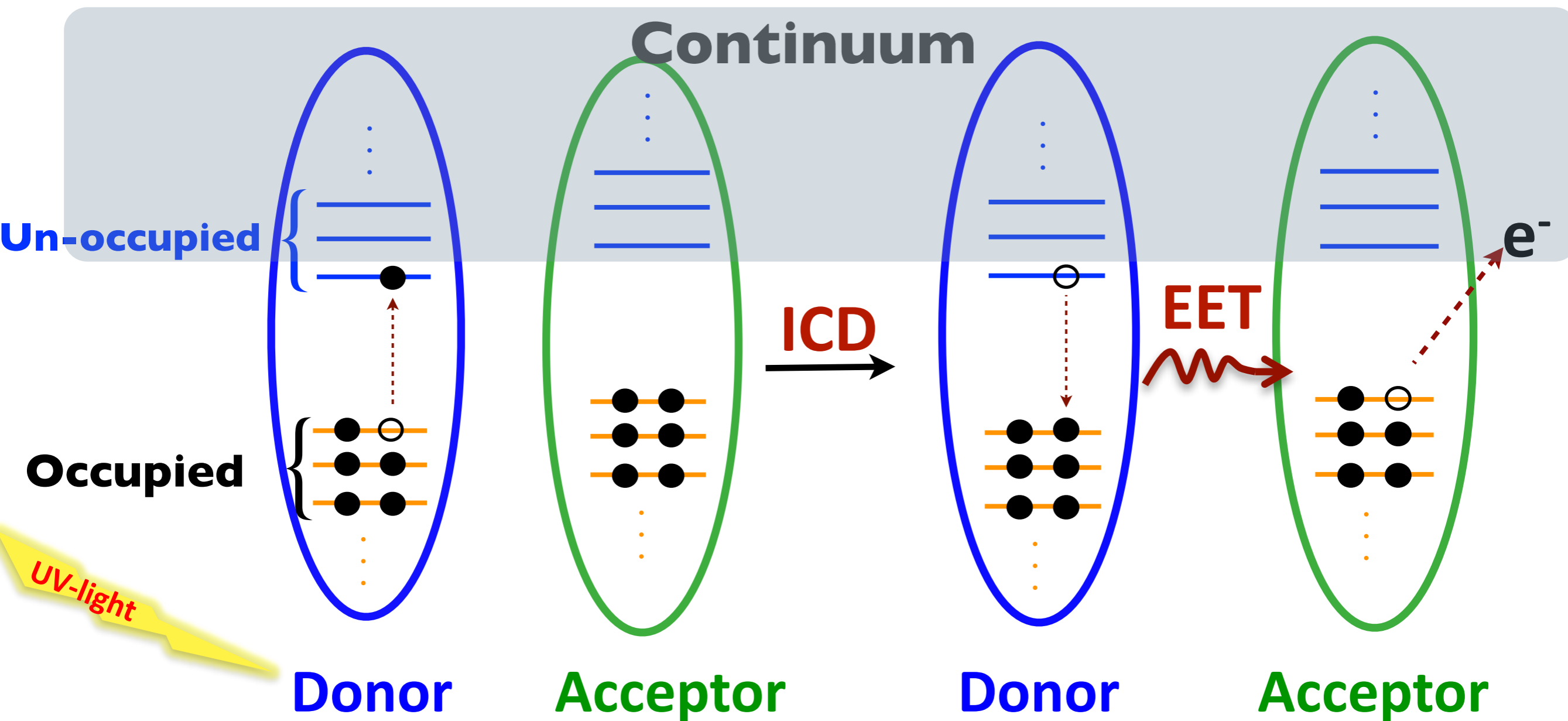


# Förster resonance energy transfer (FRET)

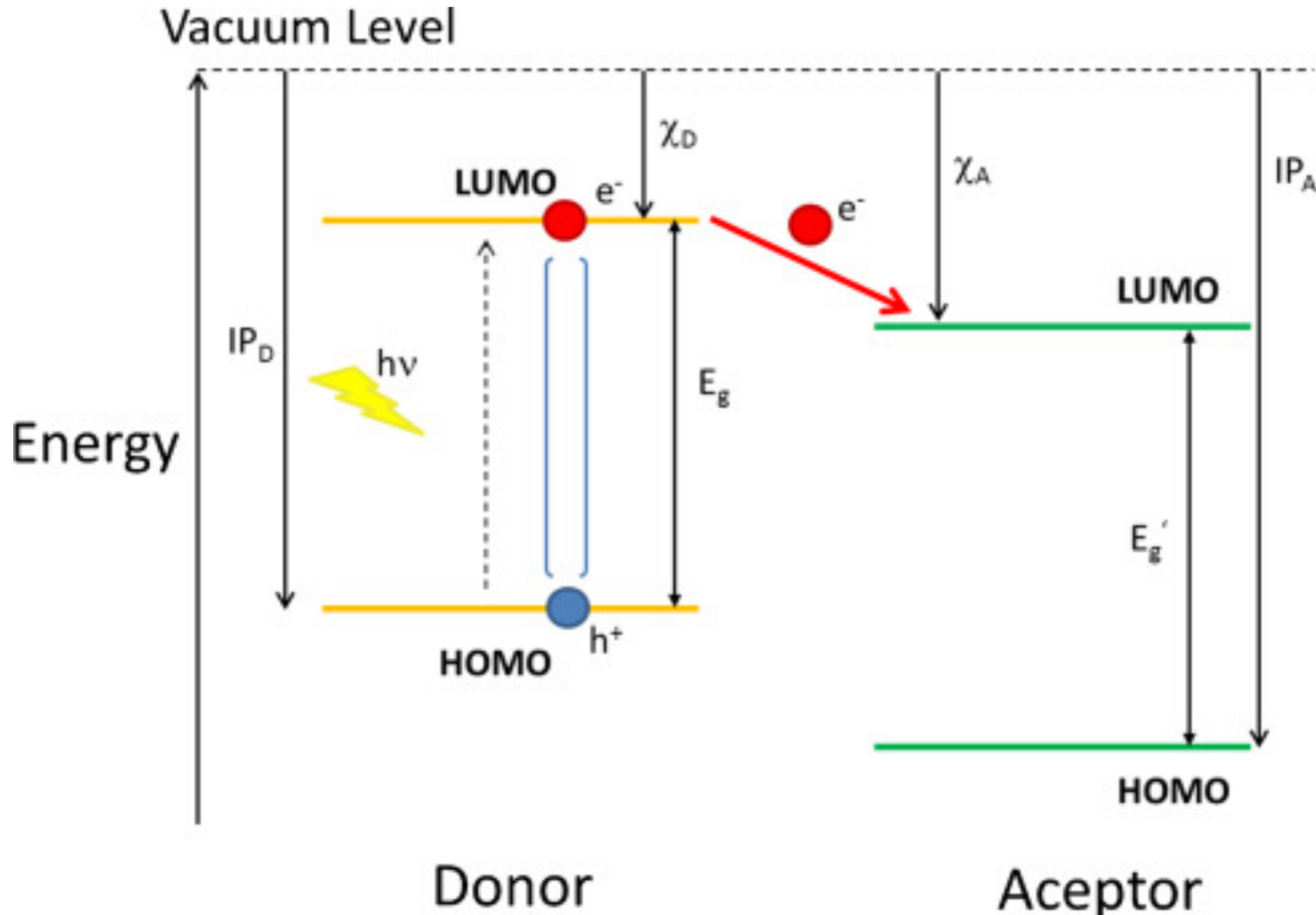




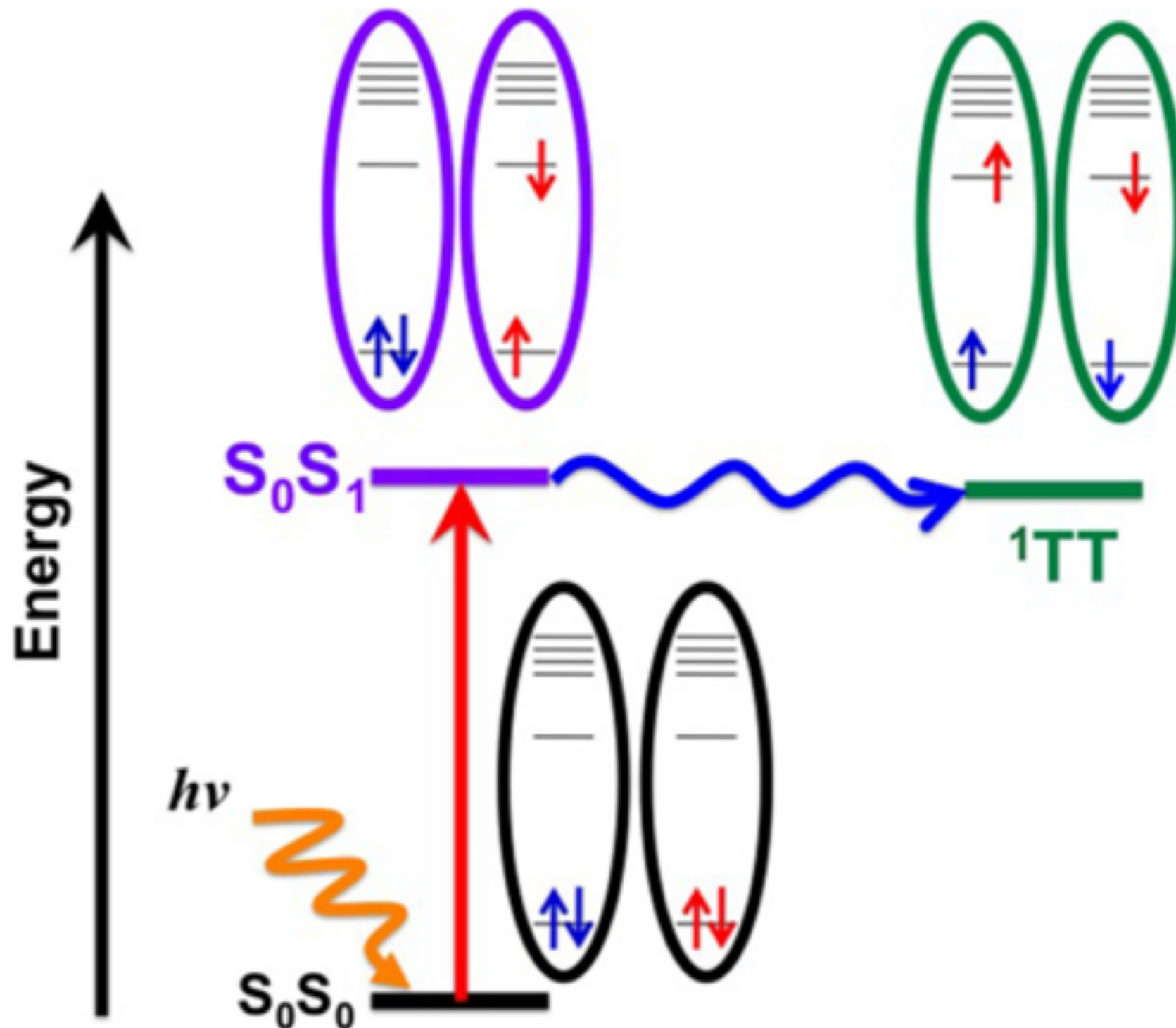
# Intermolecular Coulombic decay



# Organic photovoltaic cells



# Singlet Fission



# Photochemical processes in complex environments

## Quantum dynamics

## Quantum chemistry

- Time-dependent Schrödinger eq. for nuclei
- Multi-configuration time-dependent Hartree and its multi-layer variant
- Explicit/model potential energy surfaces
- Variational multi-configuration Gaussian (vMCG)
- Highly accurate TR spectra

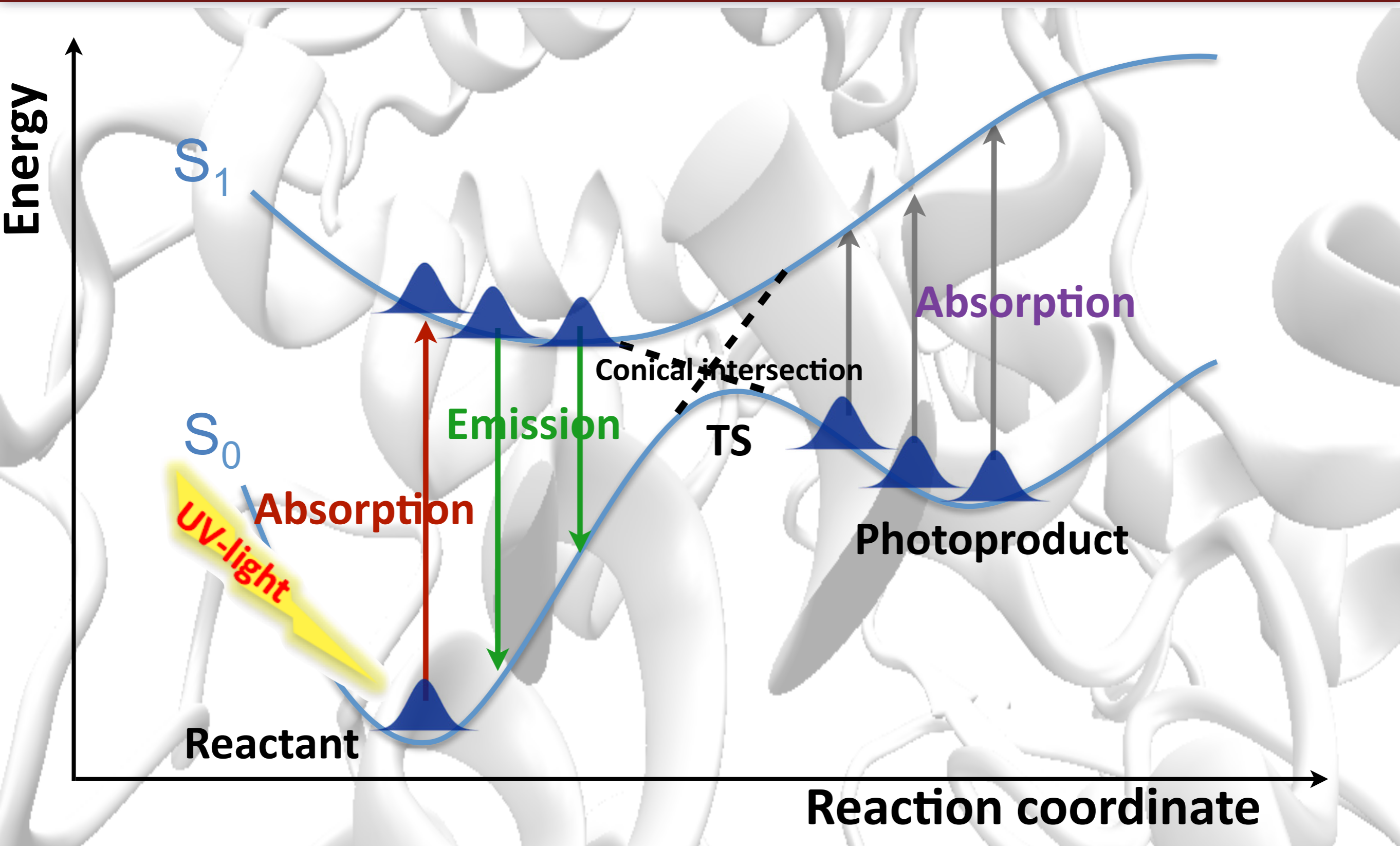
- Time-independent Schrödinger eq. for electrons
- Ground and excited electronic states methods
- Explicit and implicit environmental methods
- Quantum mechanics/molecular mechanics (QM/MM)
- QM/Effective fragment potential

# Time-dependent Schrödinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi$$

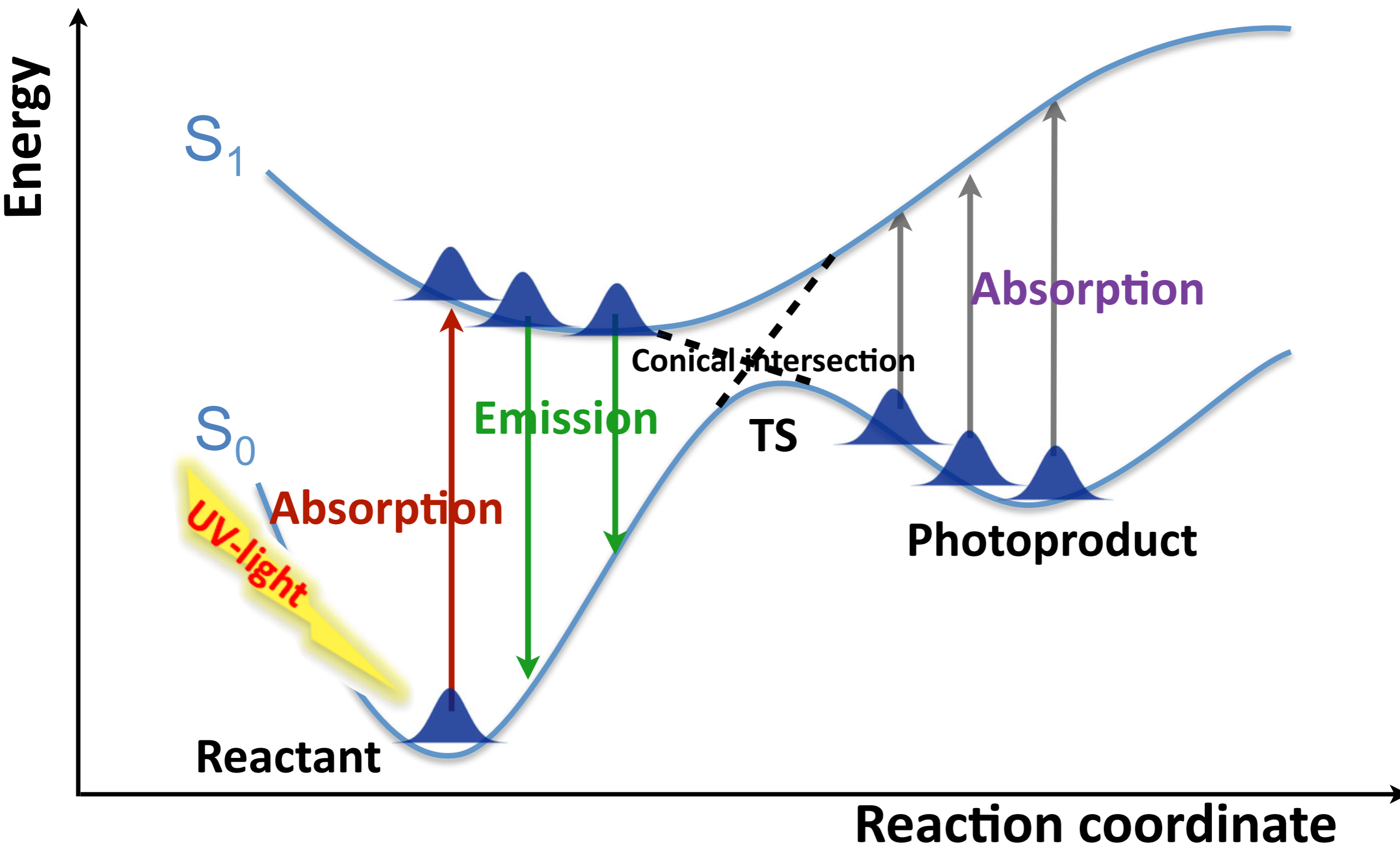


# Theoretical challenges in quantum dynamics





# Theoretical challenges in quantum dynamics



# Born-Oppenheimer approximation

- Technique to de-couple the motion of nuclei and electrons

$$H = T_e + T_N + U(r, Q)$$



**Nucleus**

**electron**



- Exact eigenstates  $\Psi(r, Q) = \sum_n \chi_n(Q) \Phi_n(r, Q)$

- Coupled equations for the expansion coefficients:

$$[T_N + V_n(Q) - E] \chi_n(Q) = \sum_m \hat{\Lambda}_{nm} \chi_m(Q)$$

- Non-adiabatic operator

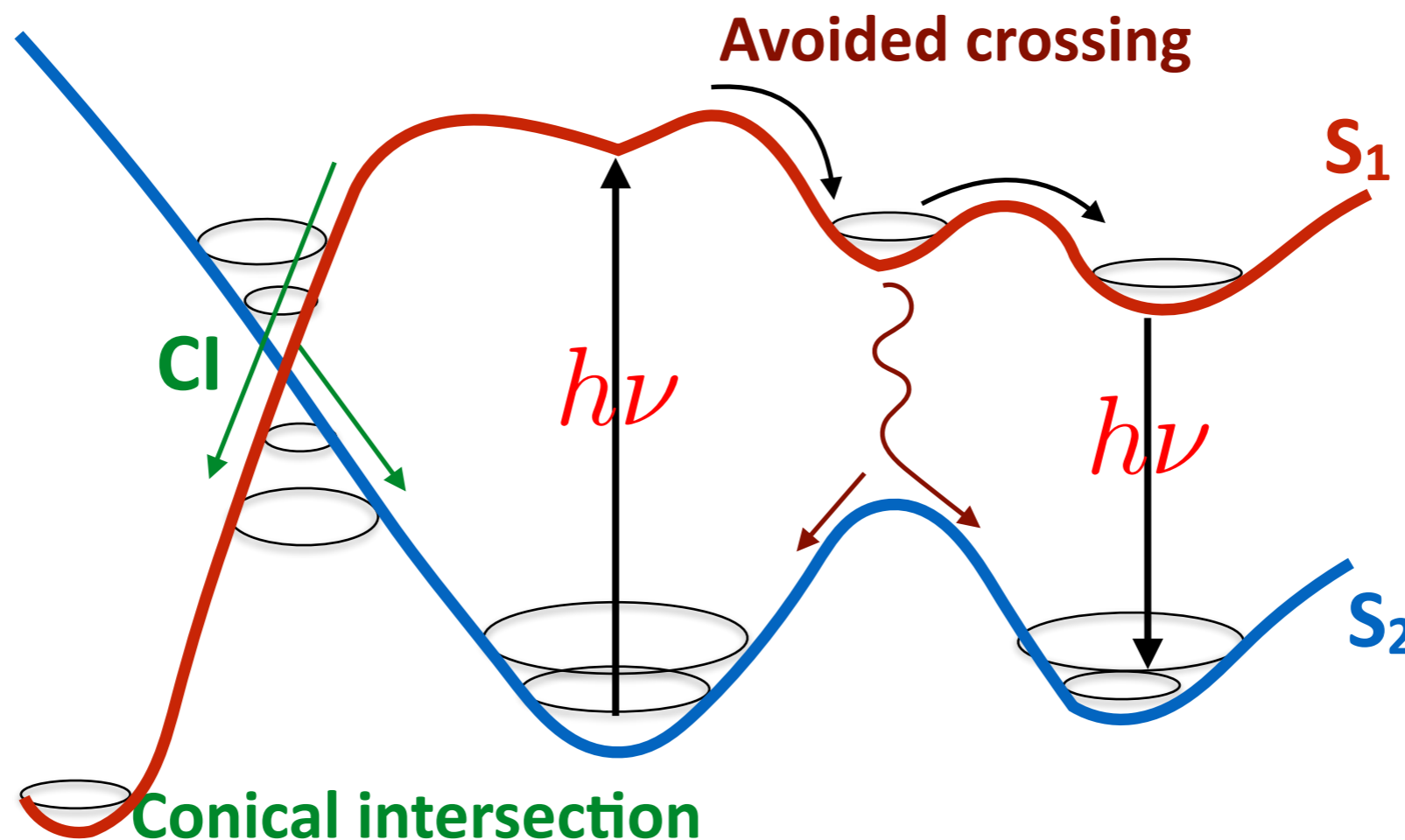
$$\begin{aligned} \hat{\Lambda}_{nm} &= \hbar^2 \sum_i \frac{1}{M_i} \langle \Phi_n | \frac{\partial}{\partial Q_i} | \Phi_m \rangle \frac{\partial}{\partial Q_i} + \langle \Phi_n | T_N | \Phi_m \rangle \\ &= \sum_{i=1}^M \frac{1}{M_i} F_{nm}^{(i)} \frac{\partial}{\partial Q_i} - G_{nm} \end{aligned}$$



# Born-Oppenheimer app. & its breakdown

- Derivative coupling terms:

$$\mathbf{F}_{nm}(\mathbf{r}, \mathbf{Q}) = \frac{\langle \Phi_n(\mathbf{r}, \mathbf{Q}) | (\nabla H_e) | \Phi_m(\mathbf{r}, \mathbf{Q}) \rangle}{V_m(\mathbf{r}, \mathbf{Q}) - V_n(\mathbf{r}, \mathbf{Q})}$$



# Conical Intersection dimension

- $3N-6$  for non-linear molecules

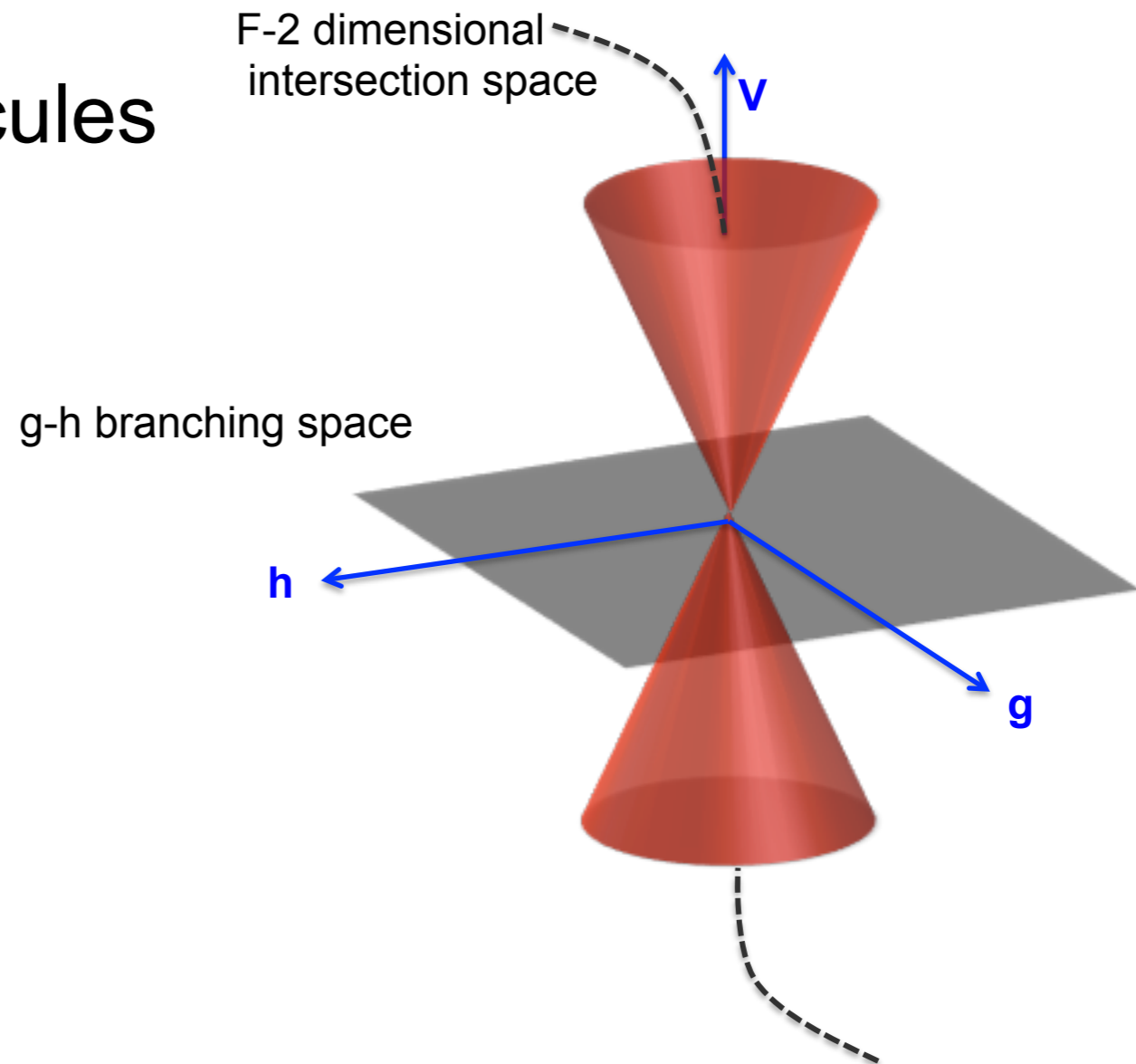
$$3N-6 - 2 = 3N - 8$$

- $3N-5$  for linear molecules

$$3N-5 - 2 = 3N - 7$$

$$N=2$$

$$3 \times 2 - 5 - 2 = -1$$



# Diabatic representation

- Replace the adiabatic functions  $\Phi(r, Q)$  by new functions  $\phi(r, Q)$

- Smooth and slowly varying functions of the nuclear coordinate

$$F_{nm} \longrightarrow 0$$

Construction by a unitary transformation of the adiabatic electronic states within a suitable subspace:

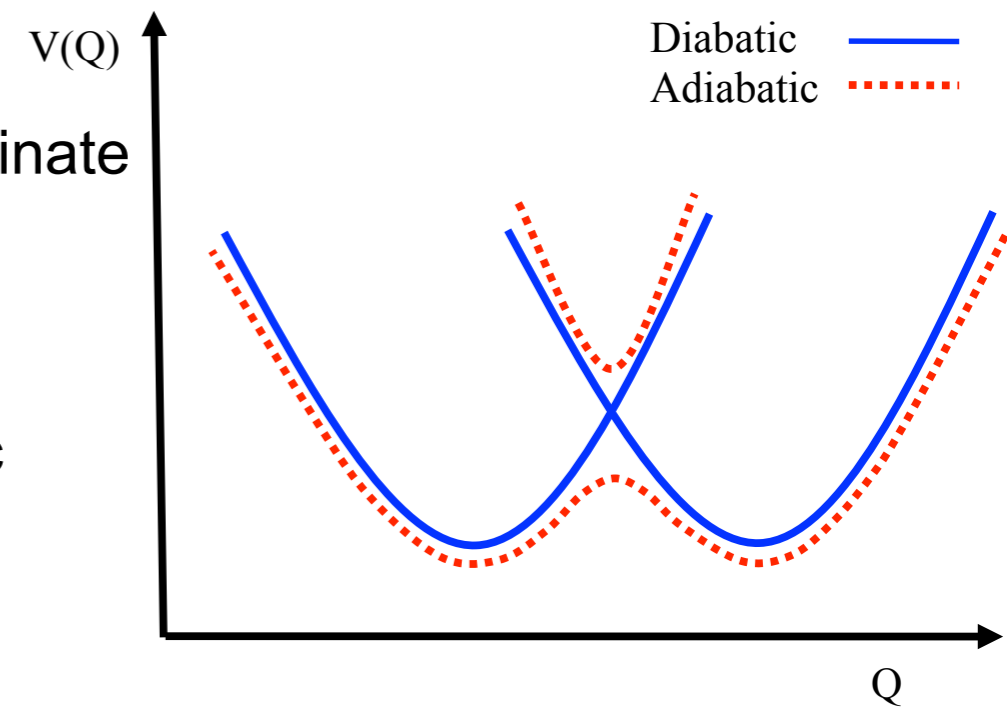
Strictly diabatic

Quasidiabatic

$$\phi^{diabatic} = \mathbf{S} \Phi^{adiabatic}$$

- Coupled equations in the diabatic representation:

$$(T_N + W_{nn}(\mathbf{Q}) - E) \tilde{\chi}_n(\mathbf{Q}) = \sum_{n \neq m} W_{nm}(\mathbf{Q}) \tilde{\chi}_m(\mathbf{Q})$$

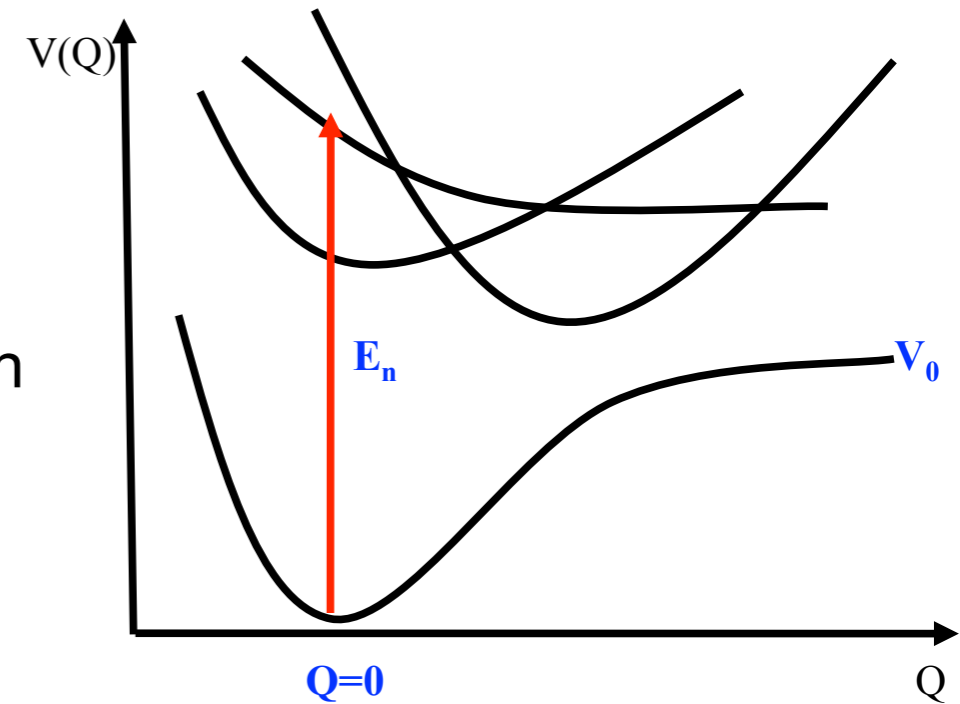


# Vibronic Coupling Model

- Matrix Hamiltonian in the diabatic basis:

$$H = T_N \mathbf{1} + \mathbf{W}(\mathbf{Q})$$

- Expanding  $W(Q)$  about a reference nuclear configuration  $Q=0$ , up to low-order terms in  $Q$



$$W_{nn}(Q) = V_0(Q) + E_n + \sum_i k_i^{(n)} Q_i + \sum_{i,j} \gamma_{ij}^{(n)} Q_i Q_j + \dots$$

$$W_{nm}(Q) = \sum_i \lambda_i^{(n,m)} Q_i + \dots \quad (n \neq m)$$

H. Köppel, W. Domcke, and L. S. Cederbaum, Adv. Chem. Phys. **57**, 59 (1984)

# Coupling constants

■ Intrastate Coupling Constant

$$\kappa_i^{(n)} = \left. \frac{\partial V_n(\mathbf{Q})}{\partial Q_i} \right|_{\mathbf{Q}=0}$$

■ Quadratic Coupling Constant

$$\gamma_i^{(n)} = \left. \frac{\partial^2 \Delta V_n(\mathbf{Q})}{\partial Q_i^2} \right|_{\mathbf{Q}=0}$$

■ Interstate Coupling Constant

$$\lambda_i^{nm} = \sqrt{\left. \frac{1}{8} \frac{\partial^2 (\Delta V_{nm})^2}{\partial Q_i^2} \right|_{\mathbf{Q}=0}}$$

■ 3 states problem

$$W_{eff}^{nm}(Q_i) = \begin{pmatrix} E_n & \lambda_i^{nm} Q_i \\ \lambda_i^{nm} Q_i & E_m \end{pmatrix}$$

$$W_{eff}^{nml}(Q_i) = \begin{pmatrix} E_n & 0 & \lambda_i^{nl} Q_i \\ 0 & E_m & \lambda_i^{ml} Q_i \\ \lambda_i^{nl} Q_i & \lambda_i^{ml} Q_i & E_l \end{pmatrix}$$

# Symmetry selection rule

- Symmetry selection rules:

$$\Gamma_n \times \Gamma_{Q_i} \times \Gamma_m \supset \Gamma_A$$

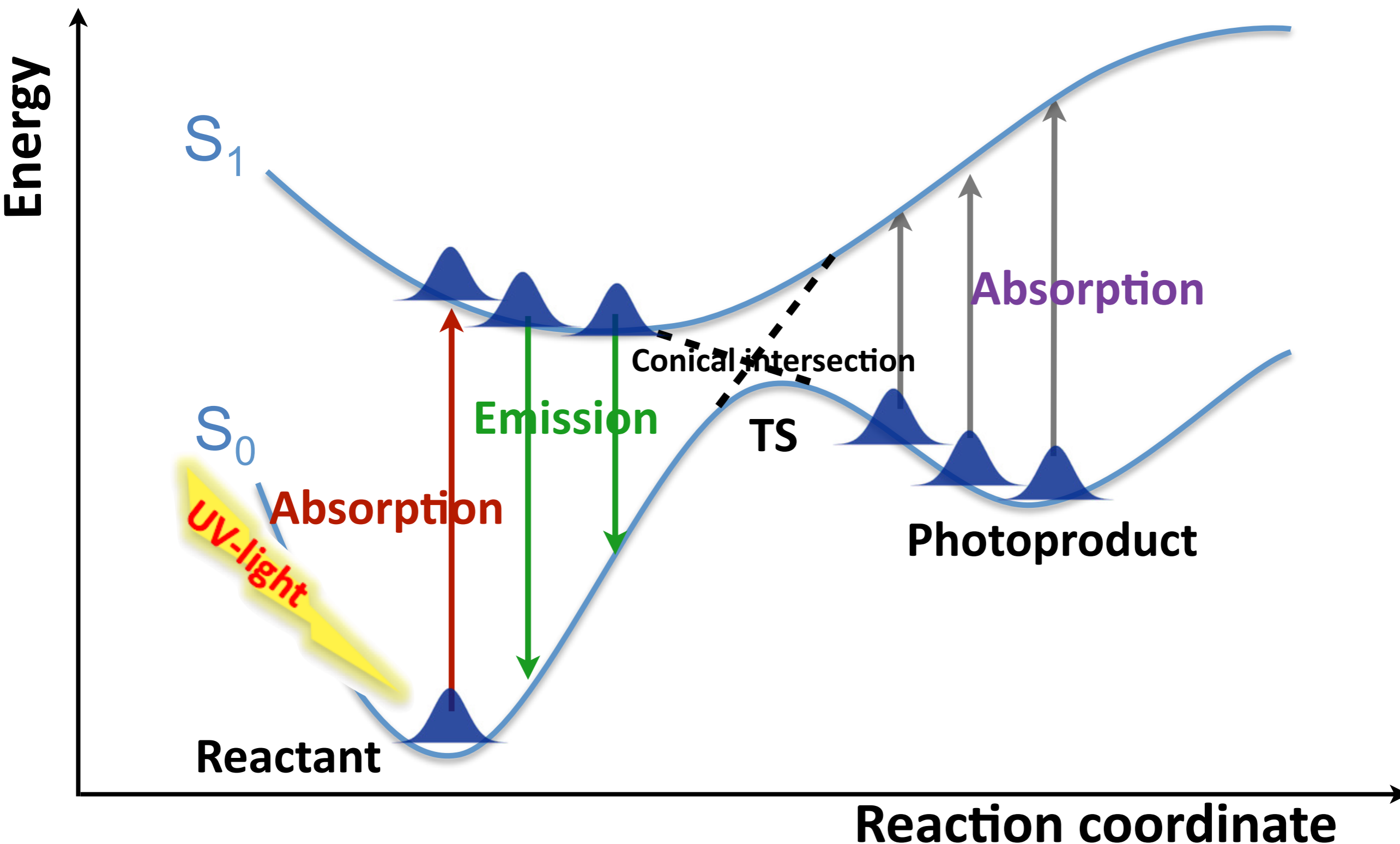
# 2 states problem

- Vibronic coupling Hamiltonian for 2 states problem with N totally symmetric and M non-totally symmetric modes

$$H = (T_N + V_0)\mathbf{1} + \begin{pmatrix} E_1 + \sum_{i=1}^N \kappa_i^{(1)} Q_{gi} & \sum_{j=1}^M \lambda_j^{(1,2)} Q_{uj} \\ \sum_{j=1}^M \lambda_j^{(1,2)} Q_{uj} & E_2 + \sum_{i=1}^N \kappa_i^{(2)} Q_{gi} \end{pmatrix}$$



# Theoretical challenges in quantum dynamics





# Vibronic-coupling dynamics

## Time-independent approach vs.

- Diagonalization of a Hamiltonian
- Eigenvalue problem

$$\hat{H} \Psi = E \Psi$$

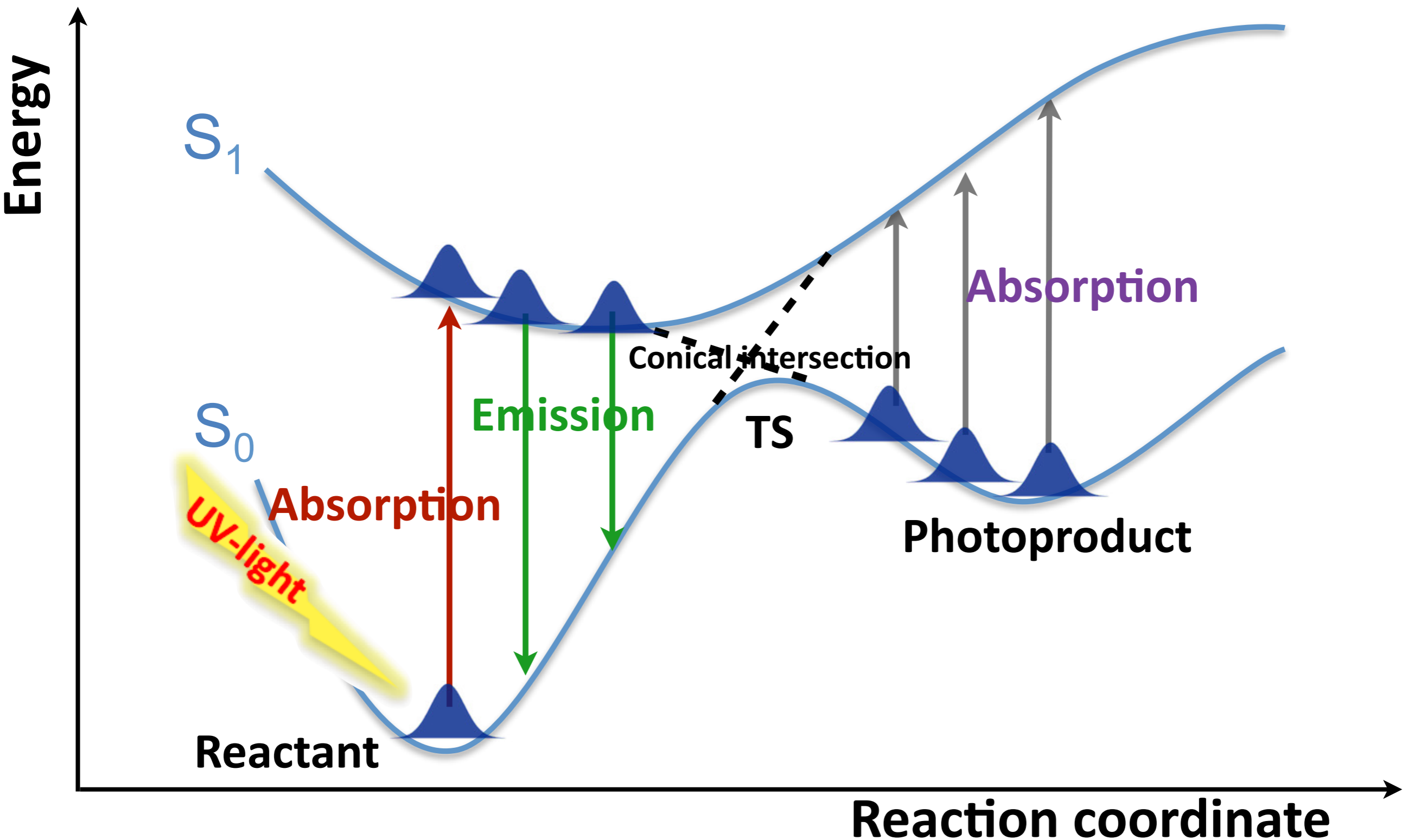
## Time-dependent approach

- Propagation of a wavepacket
- Initial value problem

$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$$

- Preparation of the initial wave packet
- Propagation of the wave packet
- Analysis of the propagated wave packet i.e. determining observable quantities

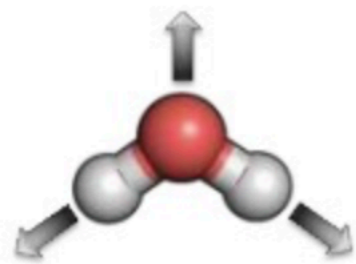
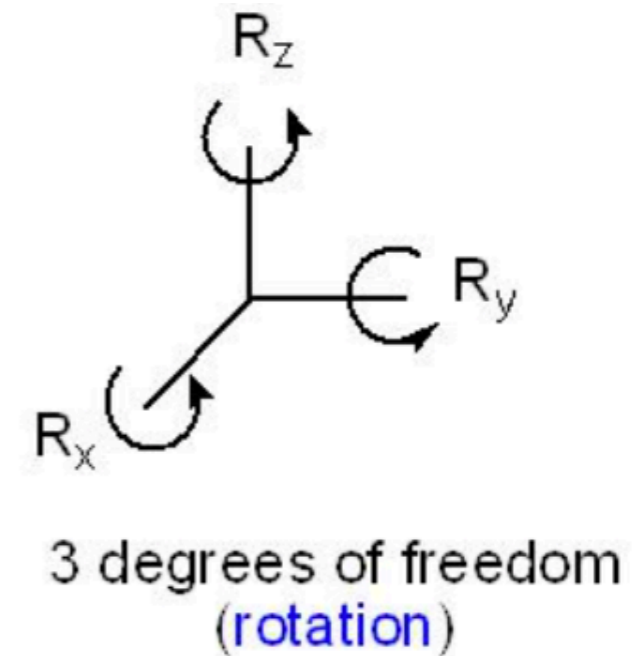
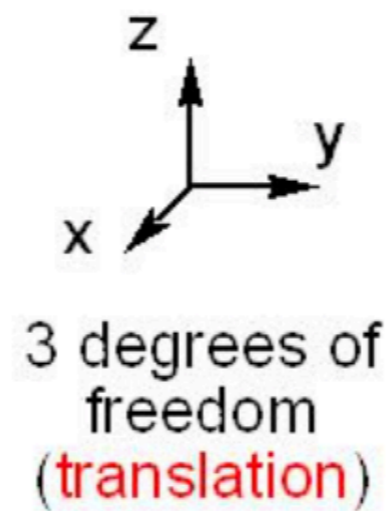
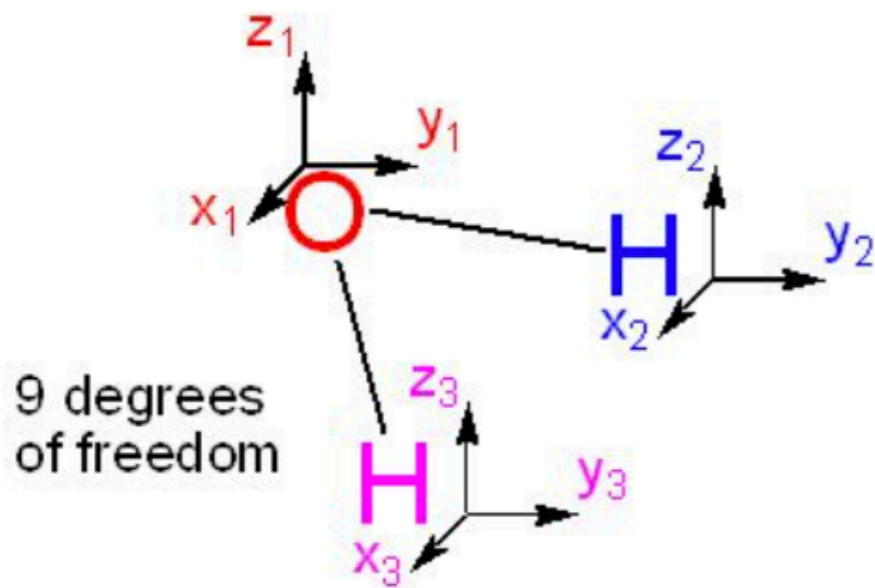
# Theoretical challenges in quantum dynamics



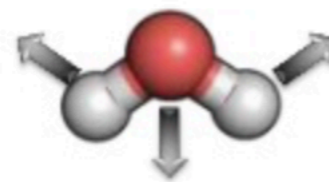
# Degrees of freedoms (DOF)

- $3N-5$  for linear molecules
- $3N-6$  for non-linear molecules

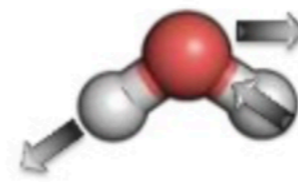
	Linear	Nonlinear
Translational	3	3
Rotational	2	3
Vibrational	$3N-5$	$3N-6$



symmetric stretch

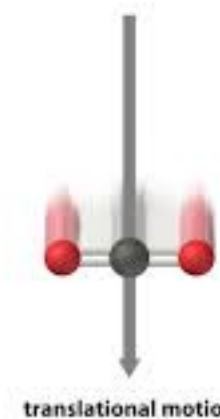
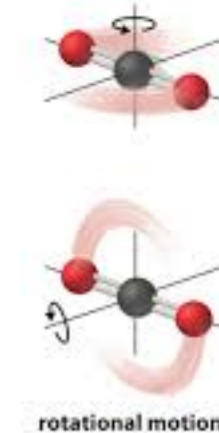
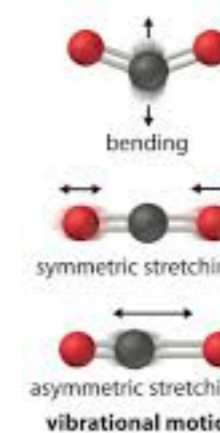


bend



asymmetric stretch

This gets large very quickly !



# Nuclear wave-packet propagation

## ■ Exact methods

$$\Psi(Q_1, \dots, Q_f, t) = \sum_{j_1}^{N_1} \dots \sum_{j_f}^{N_f} A_{j_1 \dots j_f}(t) \prod_{\kappa=1}^f \chi_{j_\kappa}^{(\kappa)}(Q_\kappa)$$

## ■ Time-dependent Hartree

$$\Psi(Q_1, \dots, Q_f, t) = A_{j_1 \dots j_f}(t) \prod_{\kappa=1}^f \chi_{j_\kappa}^{(\kappa)}(Q_\kappa, t)$$

## ■ Multi configuration time-dependent Hartree (MCTDH)

$$\Psi(Q_1, \dots, Q_f, t) = \sum_{j_1}^{n_1} \dots \sum_{j_f}^{n_f} A_{j_1 \dots j_f}(t) \prod_{\kappa=1}^f \chi_{j_\kappa}^{(\kappa)}(Q_\kappa, t)$$

U. Manthe, H.-D. Meyer, and L. S. Cederbaum, J. Chem. Phys. 97, 3199 (1992)

# Numerical effort

## Increasing accuracy

**TDH**



$$n_1 = \dots = n_f = 1$$

$$N \times f$$

**MCTDH**



$$n_k \xrightarrow{\text{approach}} N_k$$

$$n^f$$

**Standard  
 method**

$$N^f$$

Example

f=4, N=32: 6 kB

f=6, N=32: 9 kB

f=8, N=32: 12 kB

f=4, N=32, n=7: 620 kB

f=6, N=32, n=7: 32 MB

f=8, N=32, n=7: 1.03 GB

f=4, N=32: 48 MB

f=6, N=32: 48 GB

f=8, N=32: 48 TB

# Mode combination

## ■ Mode combination

$$\underbrace{(q_1, q_2, q_3, q_4, q_5)}_{Q_1}, \underbrace{(q_6, \dots, q_{f-1}, q_f)}_{Q_p}, \dots, \underbrace{(q_{f-1}, q_f)}_{Q_p}$$

$$\Psi(Q_1, \dots, Q_p, t) = \sum_{j_1}^{n_1} \dots \sum_{j_p}^{n_p} A_{j_1 \dots j_p}(t) \prod_{\kappa=1}^p \varphi_{j_\kappa}^{(\kappa)}(Q_\kappa, t)$$

## ■ Single Particle Functions (SPF)

$$\varphi_{j_\kappa}^{(\kappa)}(Q_\kappa, t) = \sum_{l_1=1}^{N_{1,\kappa}} \dots \sum_{l_d=1}^{N_{d,\kappa}} c_{j_\kappa l_1 \dots l_d}^{(\kappa)}(t) \chi_{l_1}^{(\kappa)}(q_{1,\kappa}) \dots \chi_{l_d}^{(\kappa)}(q_{d,\kappa})$$

# MCTDH equation of motion

- Constrains

$$\langle \varphi_j^{(\kappa)}(0) | \varphi_l^{(\kappa)}(0) \rangle = \delta_{jl}$$

$$\langle \varphi_j^{(\kappa)}(t) | \dot{\varphi}_l^{(\kappa)}(t) \rangle = -i \langle \varphi_j^{(\kappa)}(t) | g^{(\kappa)} | \varphi_l^{(\kappa)}(t) \rangle$$

Constrain operator

- Time-dependent variational principle (Dirac Frenkel VP)

$$\langle \delta \Psi | \hat{H} - i \frac{\partial}{\partial t} | \Psi \rangle = 0$$

$$i \dot{A}_J = \sum_L \langle \Phi_J | \hat{H} | \Phi_L \rangle A_L$$

$$i \dot{\varphi}_j^{(\kappa)} = \left( 1 - P^{(\kappa)} \right) \sum_{k,l} \rho_{j,k}^{(\kappa)-1} \langle \hat{\mathbf{H}} \rangle_{k,l}^{(\kappa)} \varphi_l^{(\kappa)}$$

# Product representation of H

- The computation of **Hamiltonian matrix elements** and **mean-fields** requires the evaluation of multi-dimensional integrals.
- These multi-dimensional integrals may be written as a product of one-dimensional integrals, if the Hamiltonian satisfies the following product form:

$$\hat{H} = \sum_{r=1}^s c_r \prod_{\kappa=1}^p \hat{h}_r^{(\kappa)}$$

$$\langle \Phi_J | \hat{H} | \Phi_L \rangle = \sum_{r=1}^s c_r \langle \varphi_{j_1}^{(1)} | \hat{h}_r^{(1)} | \varphi_{l_1}^{(1)} \rangle \cdots \langle \varphi_{j_p}^{(p)} | \hat{h}_r^{(p)} | \varphi_{l_p}^{(p)} \rangle$$

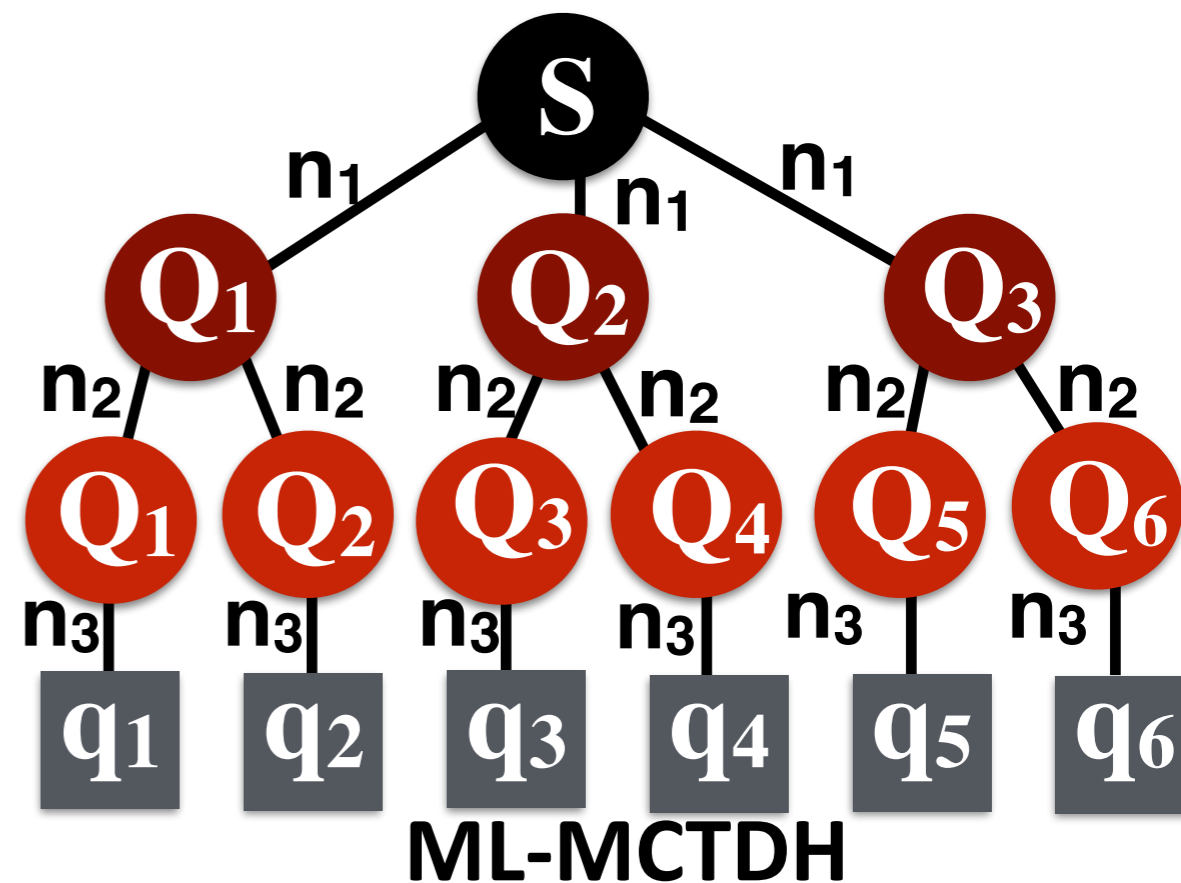
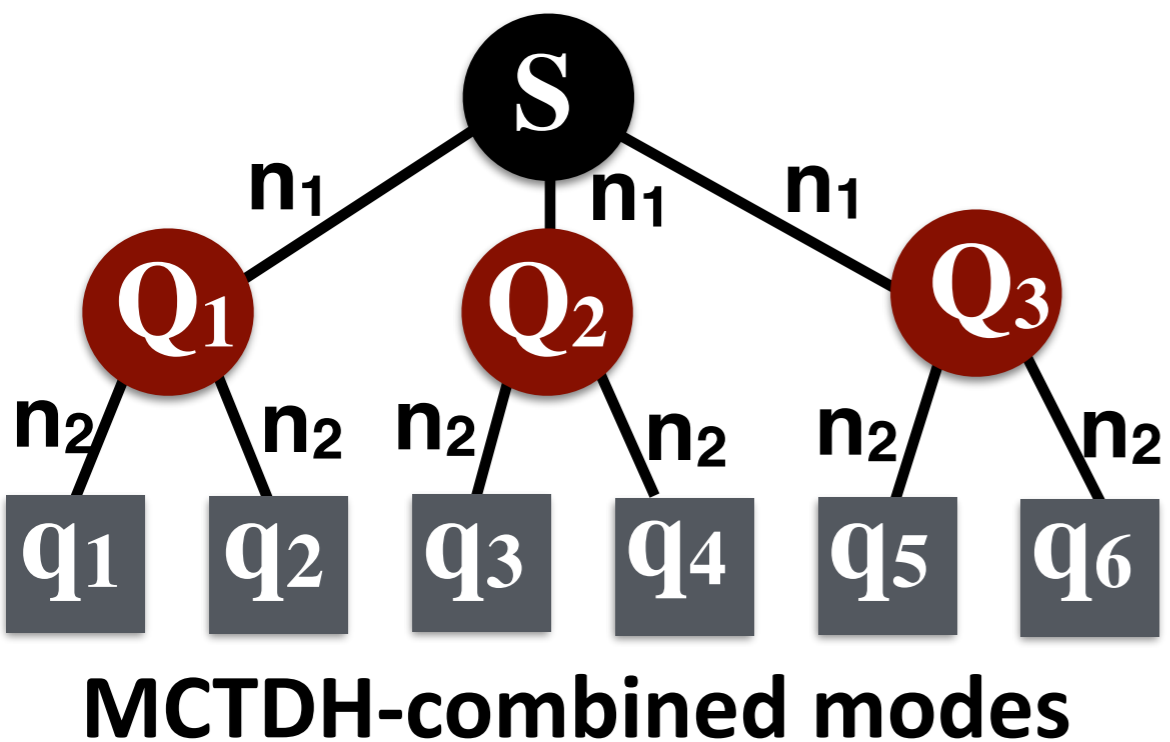
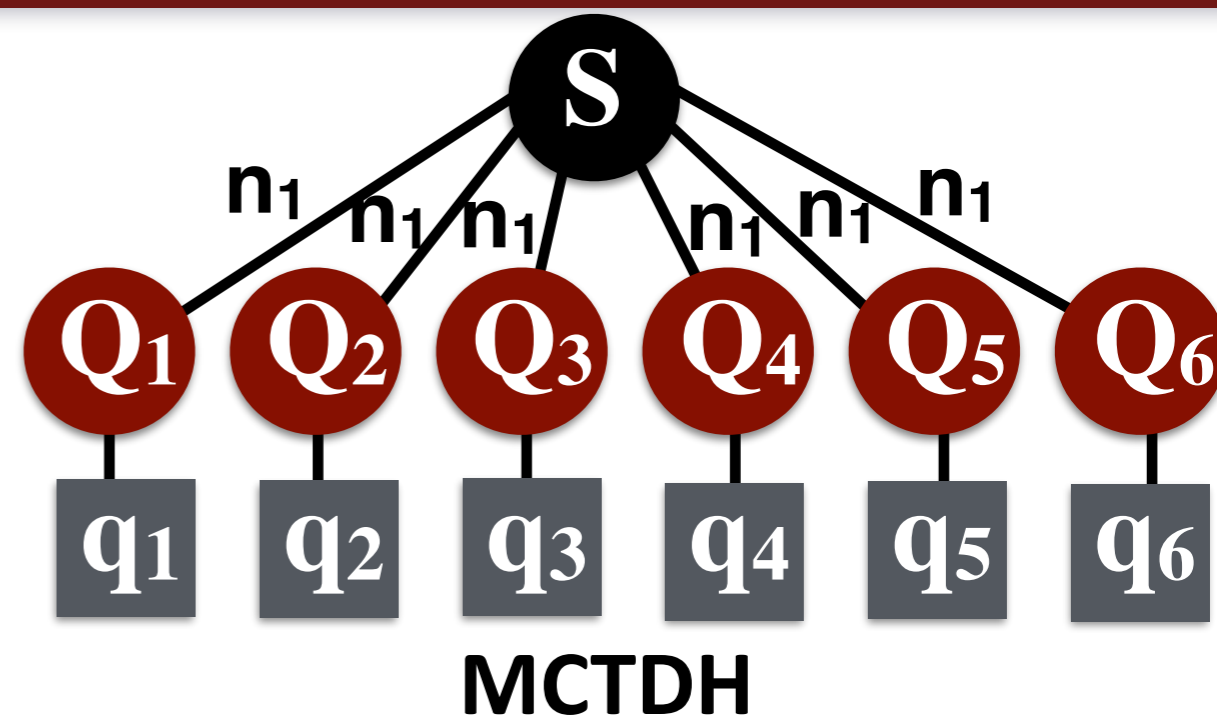
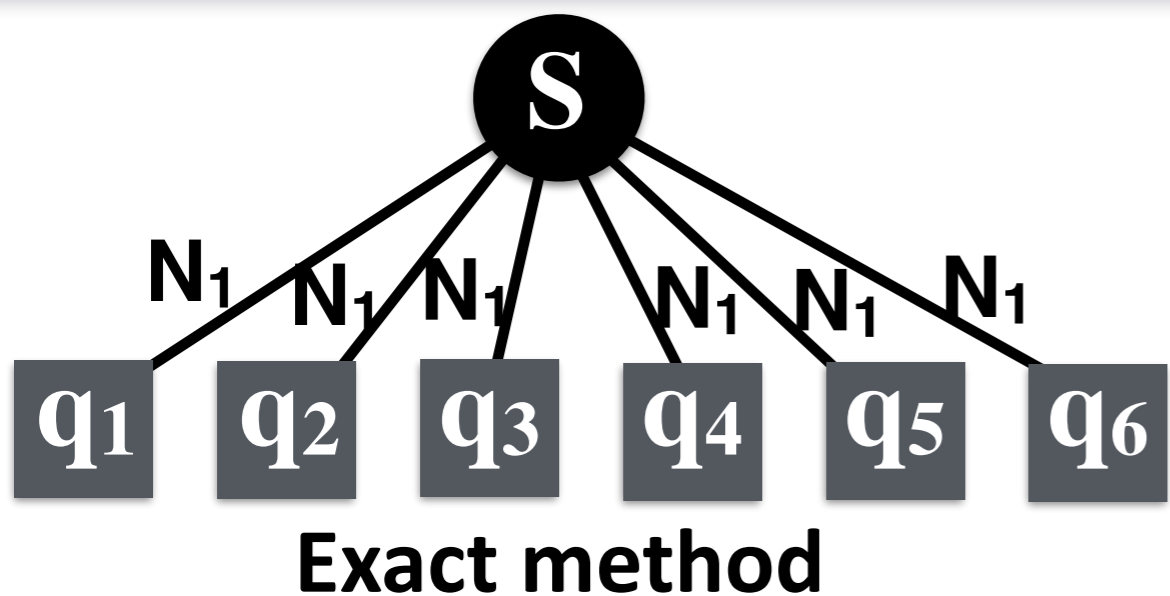
M.H. BECK, A. JÄCKLE, G. A. WORTH, and H.-D. MEYER. *Phys. Rep.*, 324:1, 2000



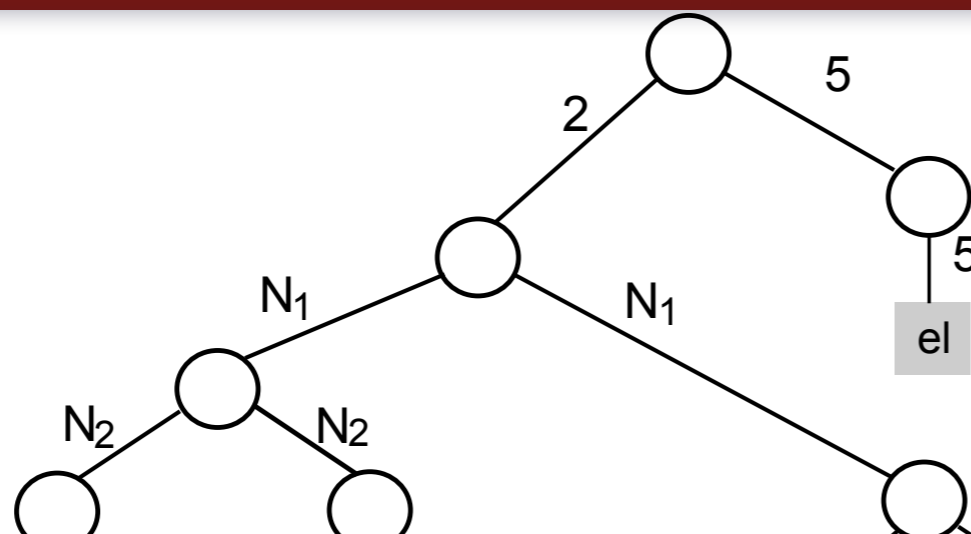
# Non-adiabatic problems: Multi-set algorithm

- Single-set vs **Multi-set** formulation

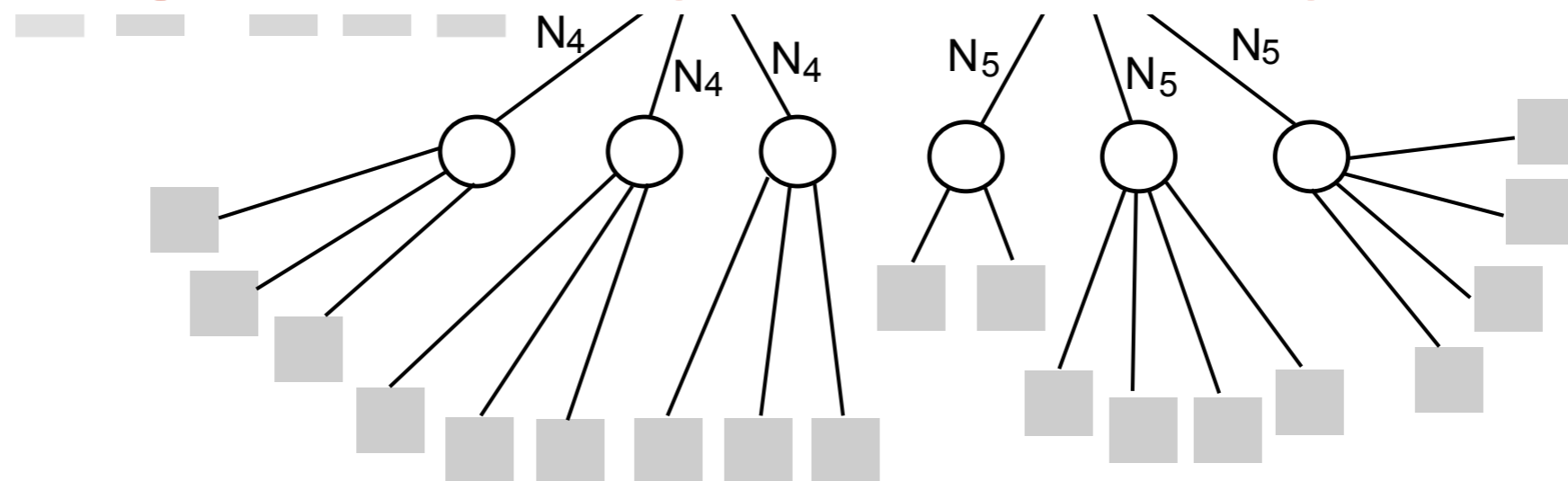
$$\Psi(Q_1, \dots, Q_p, t) = \sum_{\alpha=1}^{\alpha} \sum_{j_1}^{n_1^{\alpha}} \dots \sum_{j_p}^{n_p^{\alpha}} A_{j_1 \dots j_p}^{\alpha}(t) \prod_{\kappa=1}^p \varphi_{j_{\kappa}}^{(\alpha, \kappa)}(Q_{\kappa}, t) |\alpha\rangle$$
$$= \sum_{\alpha} \sum_J A_J^{(\alpha)} \Phi_J^{(\alpha)} |\alpha\rangle$$



# Pyrrole 24D, ML-MCTDH-tree



## Coarse grained quantum dynamics



MCTDH (89 h)

ML-MCTDH (15 min)

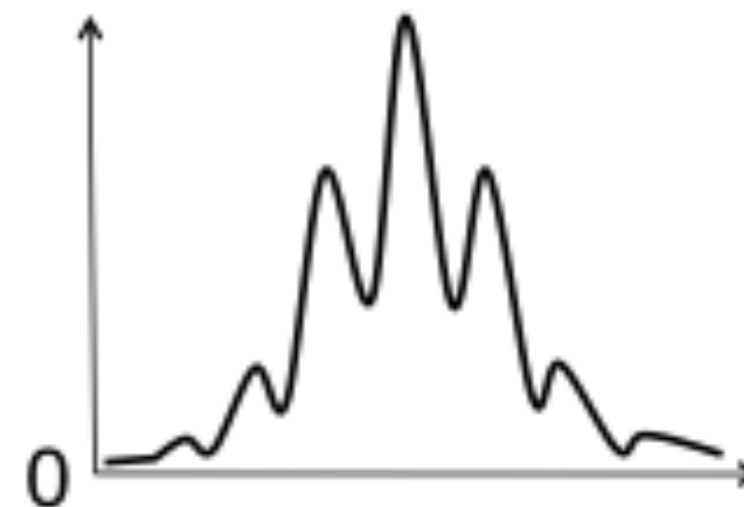
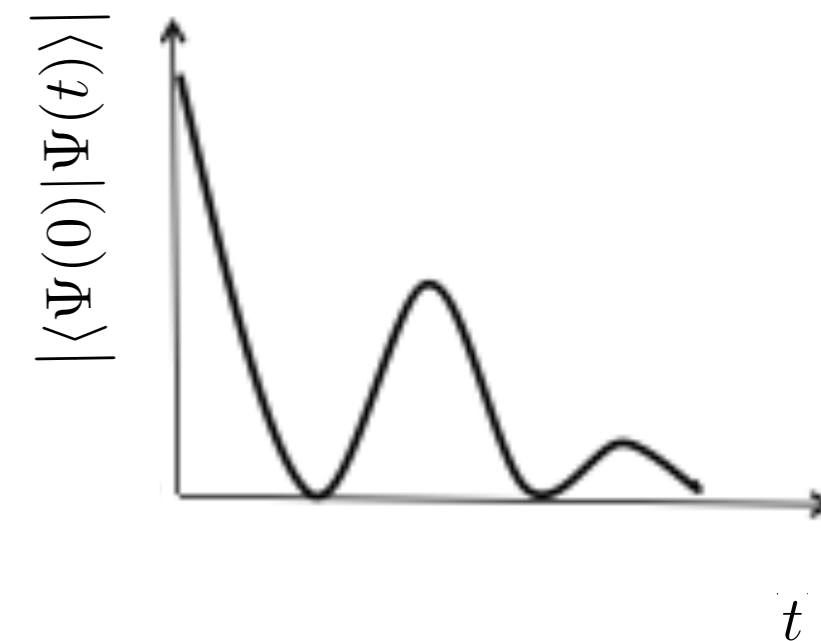
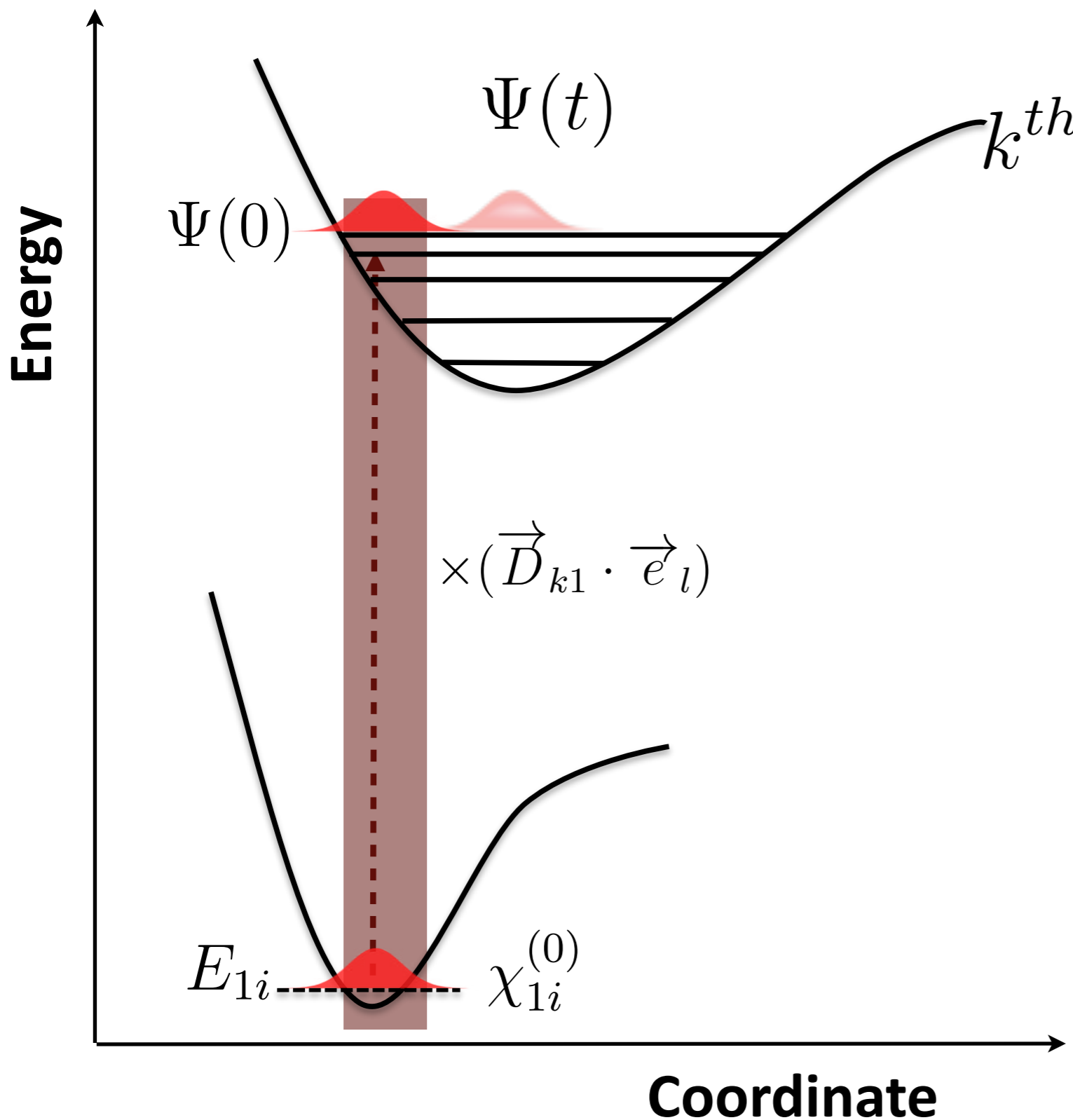
S. Faraji et al., *J. Chem. Phys.* 135, 154310, **2011**.

# Dynamical observables

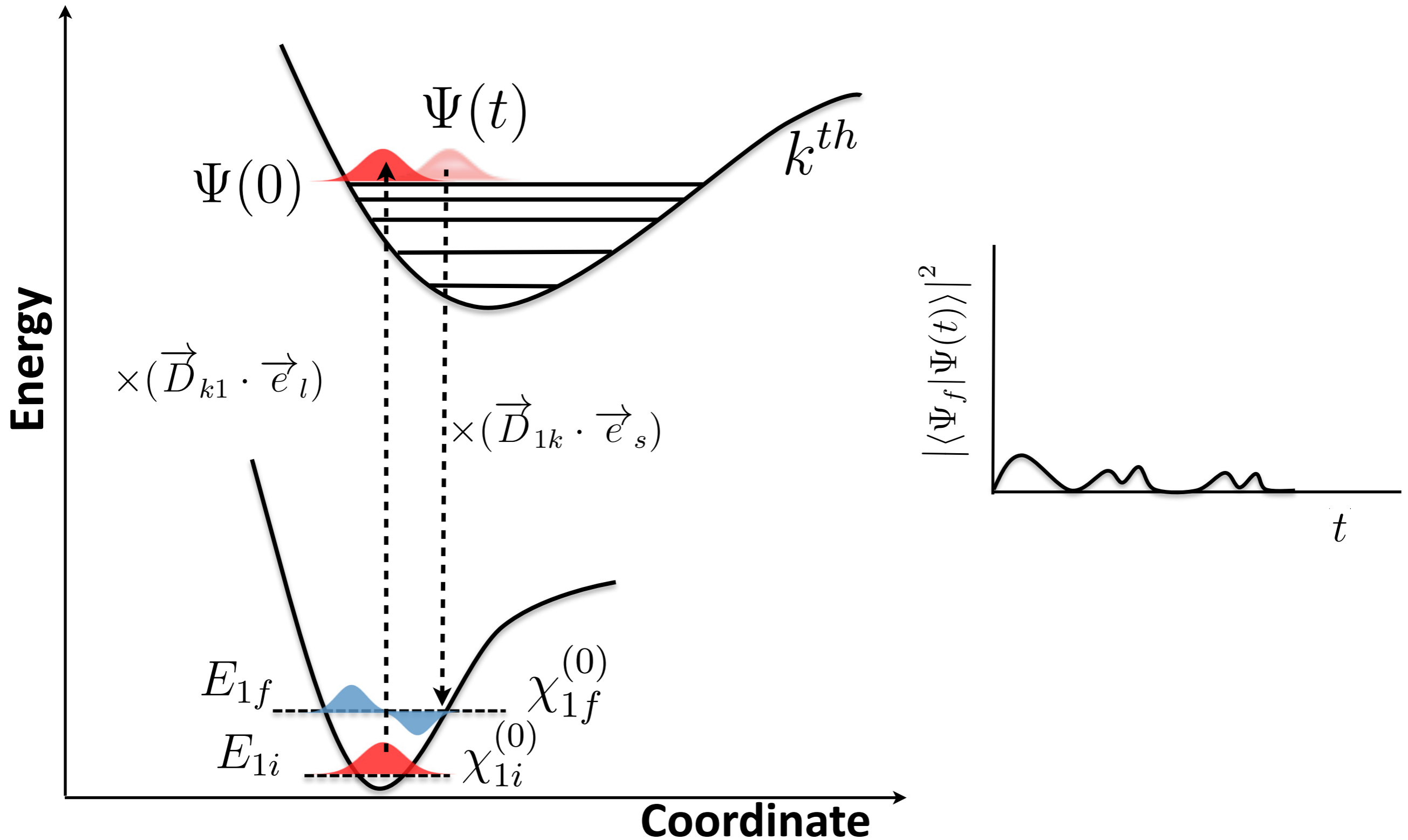
- Various spectra
- Electronic population
- Flux into a reaction channel
- Probability density along a coordinate
- .....



# Absorption spectrum



# Resonance Raman spectrum

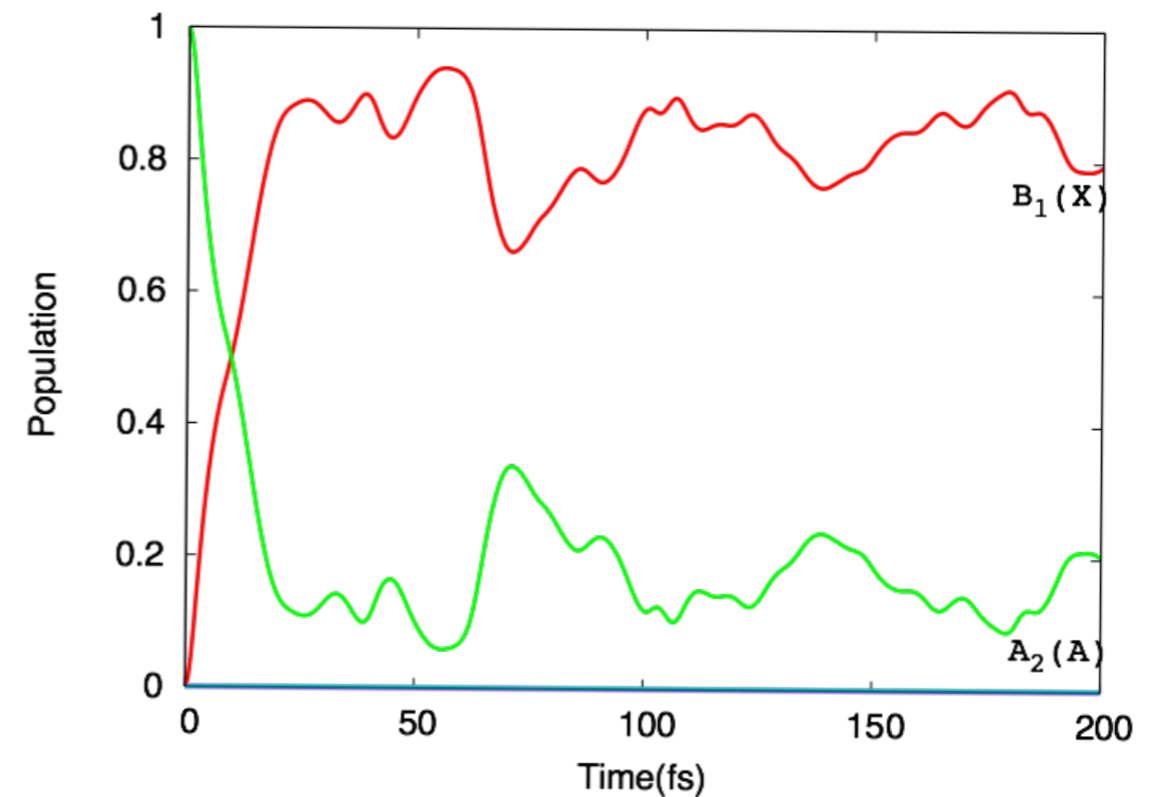
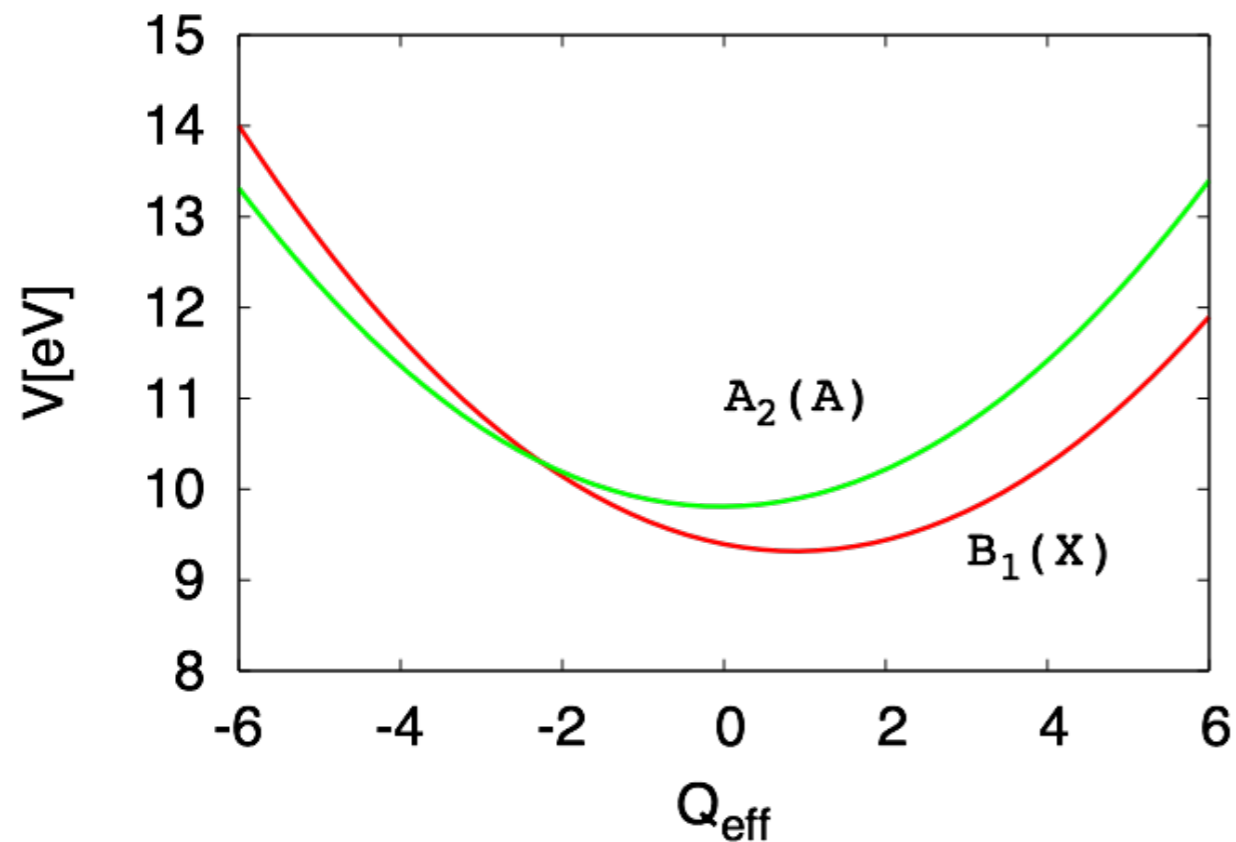


# Diabatic electronic population

- Probability of the wavepacket being on one of the electronic surfaces.

$$P_{\alpha}^{(d)} = \|\Psi_{\alpha}\|^2$$

- Fluorescence dynamics



# Current state of quantum dynamics

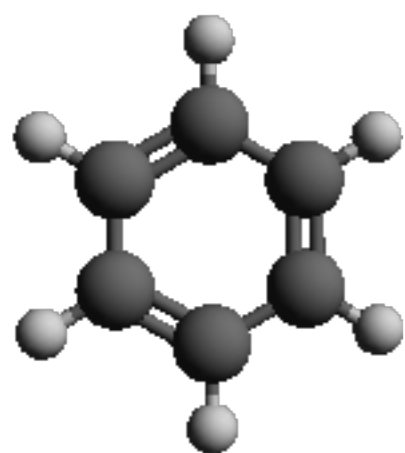
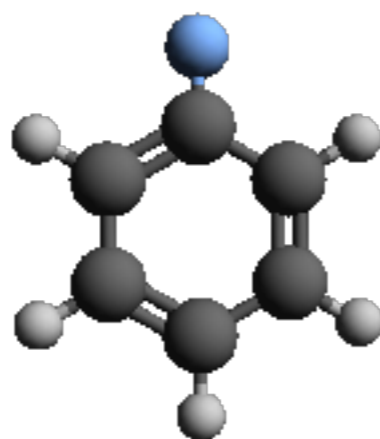
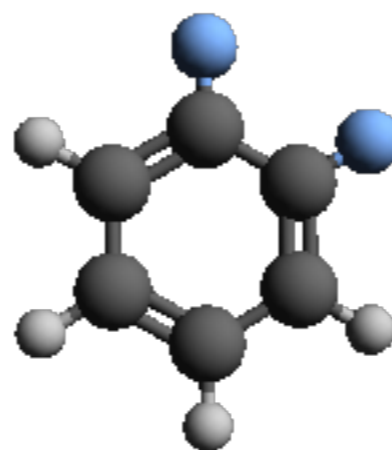
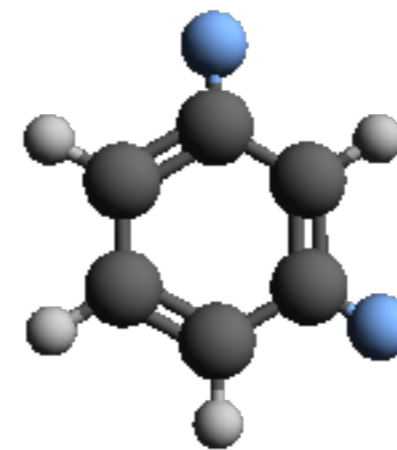
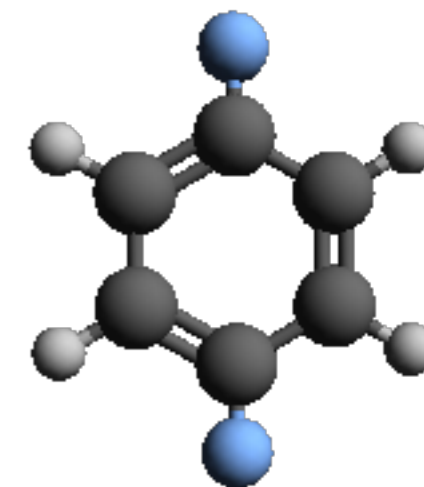
## Collection of MCTDH calculations on vibrance coupling model potential

Process	System	Formula	$f$	$e$
Photo-excitation	pyrazine	$C_4H_4N_2$	24	2
	furan	$C_4H_4O$	13	4
	pyrrole	$C_4H_5N$	10 (24)	5
Photo-ionization	butatriene	$C_4H_4^+$	18	2
	allene	$C_3H_4^+$	15	3
	pentatetraene	$C_5H_4^+$	21	3
	benzene	$C_6H_6^+$	13	5
	monofluorobenzene	$C_6F_2H_5^+$	12(30)	5
	difluorobenzene	$C_6F_2H_4^+$	12(30)	5
	trifluorobenzene	$C_6F_2H_4^+$	12(30)	5
	cyclopropane	$C_3H_6^+$	14	4
	trifluoroacetonitrile	$CF_3CN^+$	12	5
	phenylacetylene	$C_8H_6^+$	24	4
	naphthalene	$C_{10}H_6^+$	29	6
anthracene	$C_{14}H_{10}^+$	31	6	
Photo-detachment	phenide	$C_6H_5$	27	2
	nitrate radical	$NO_3$	4	2



# Fluorinated benzene radical cations

## Fluorescence

 $D_{6h}$  $C_{2v}$  $C_{2v}$   
Ortho $C_{2v}$   
Meta $D_{2h}$   
Para

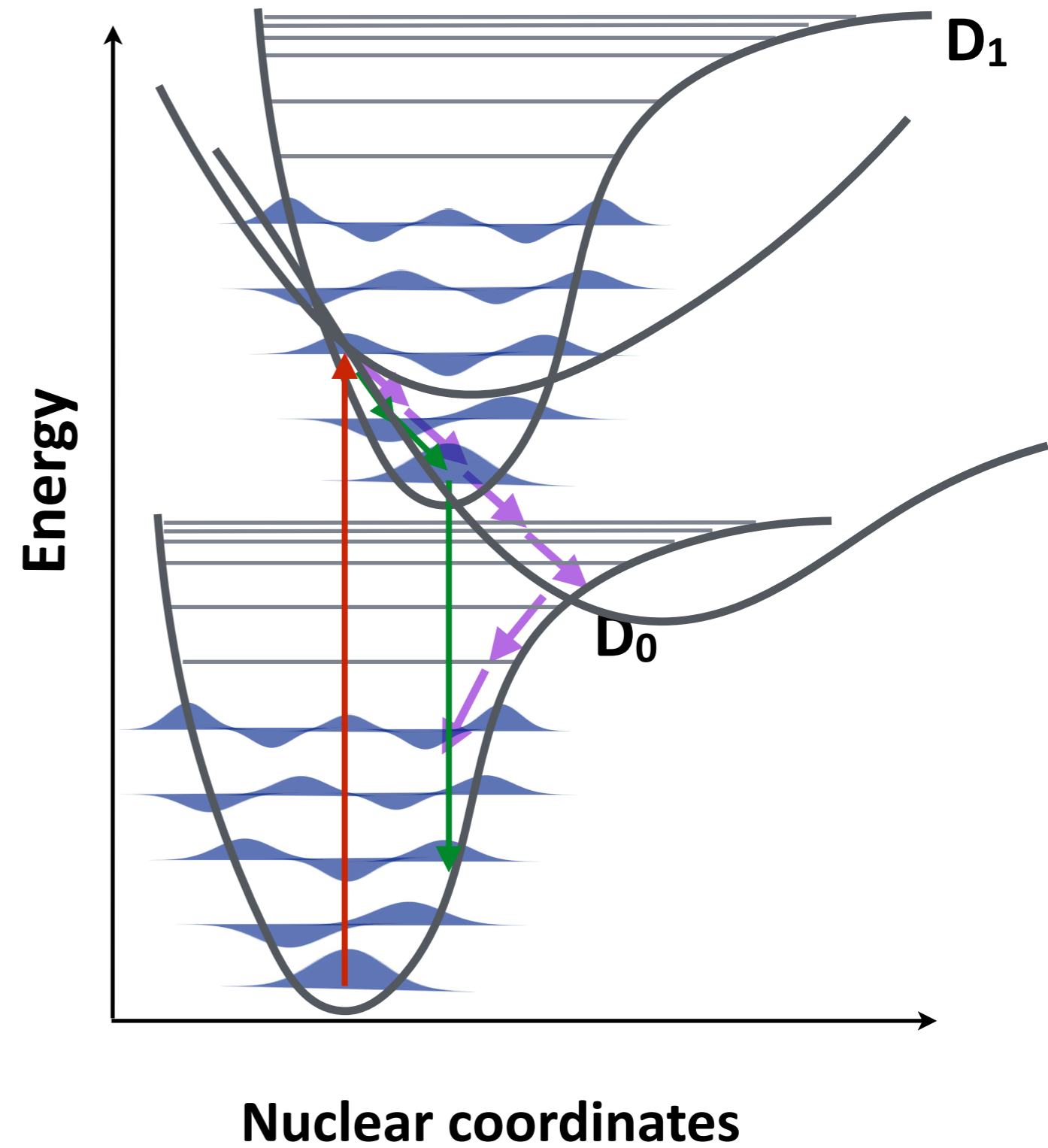
S. Faraji, H. Köppel, *J. Chem. Phys.* 129, 074310, **2008**.

S. Faraji, H-D. Meyer, H. Köppel, *J. Chem. Phys.* 129, 074311, **2008**.

S. Faraji, E. Gindensperger, H. Köppel, *Chem. Phys.*, 97, 239, **2010**.

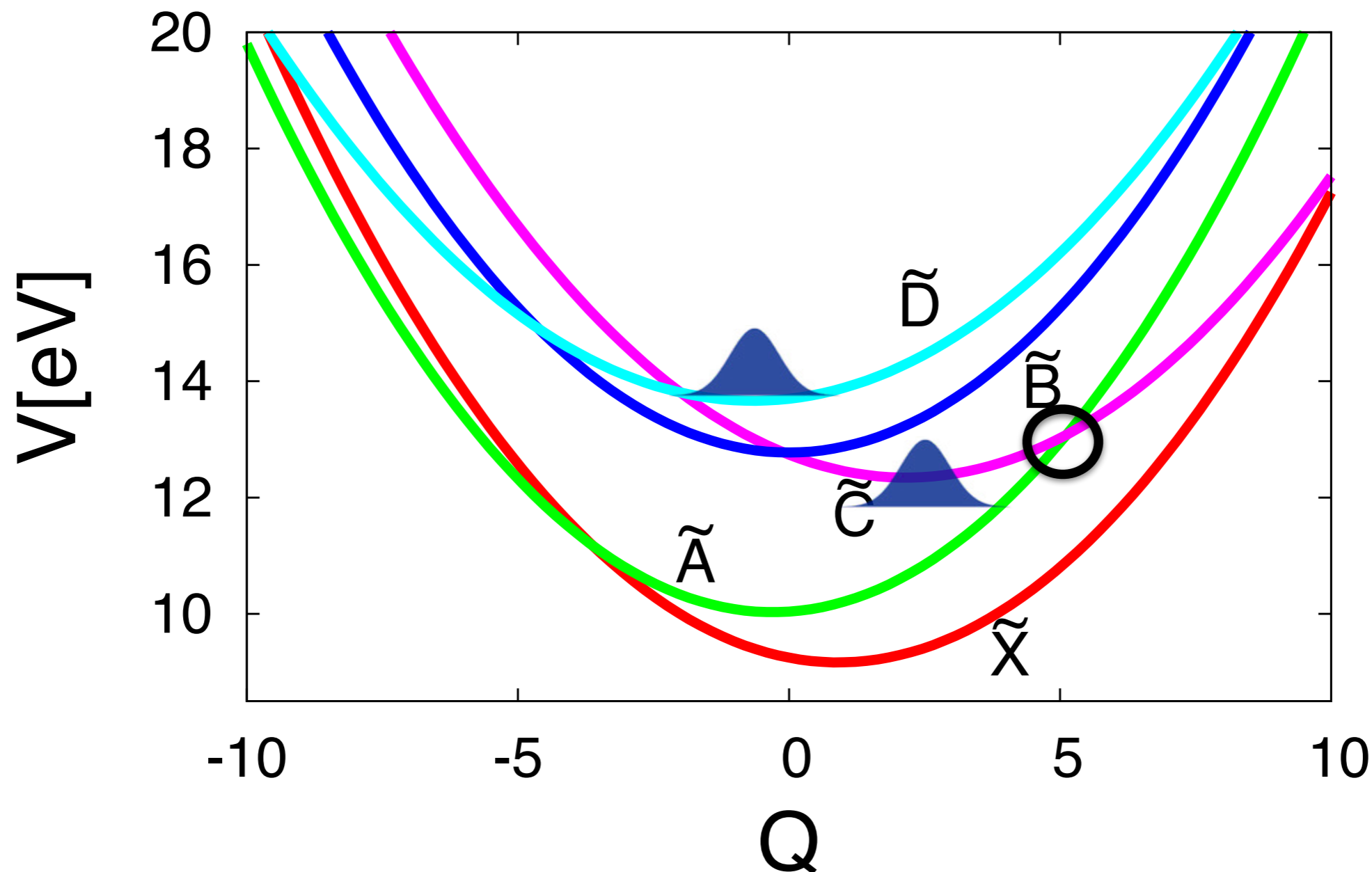
# Radiative vs. non-radiative decays

- Fluorescence
- Internal conversion





# Vibronic coupling model PESs and minima of conical intersection seams



**Non-Fluorescent**  
**Fluorescent**  
**Non-Fluorescent**

**Ortho**  
**Meta**  
**Para**

**13.07 eV**  
**13.65 eV**  
**13.08 eV**



# Minima of conical intersection seams

## Ortho

	$\tilde{X}$	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\tilde{D}$
$\tilde{X}$	9.14 9.13	9.62	25.63	13.92	13.64
$\tilde{A}$		9.62 9.61	28.97	13.07	13.57
$\tilde{B}$			12.66 12.62	12.70	12.96
$\tilde{C}$				12.44 12.49	13.05
$\tilde{D}$					12.87 12.91

## Para

	$\tilde{X}$	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\tilde{D}$
$\tilde{X}$	8.96 8.96	9.97	31.13	15.53	19.53
$\tilde{A}$		9.87 9.86	23.98	13.08	19.29
$\tilde{B}$			12.60 12.59	12.62	14.31
$\tilde{C}$				12.30 12.30	13.65
$\tilde{D}$					13.58 13.56

## Meta

	$\tilde{X}$	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\tilde{D}$
$\tilde{X}$	9.19 9.18	9.62	21.47	14.68	13.84
$\tilde{A}$		9.61 9.61	21.13	13.65	14.18
$\tilde{B}$			12.61 12.62	12.74	13.09
$\tilde{C}$				12.60 12.59	13.16
$\tilde{D}$					13.08 13.13

# Time-dependent electronic populations

**Non- fluorescent**

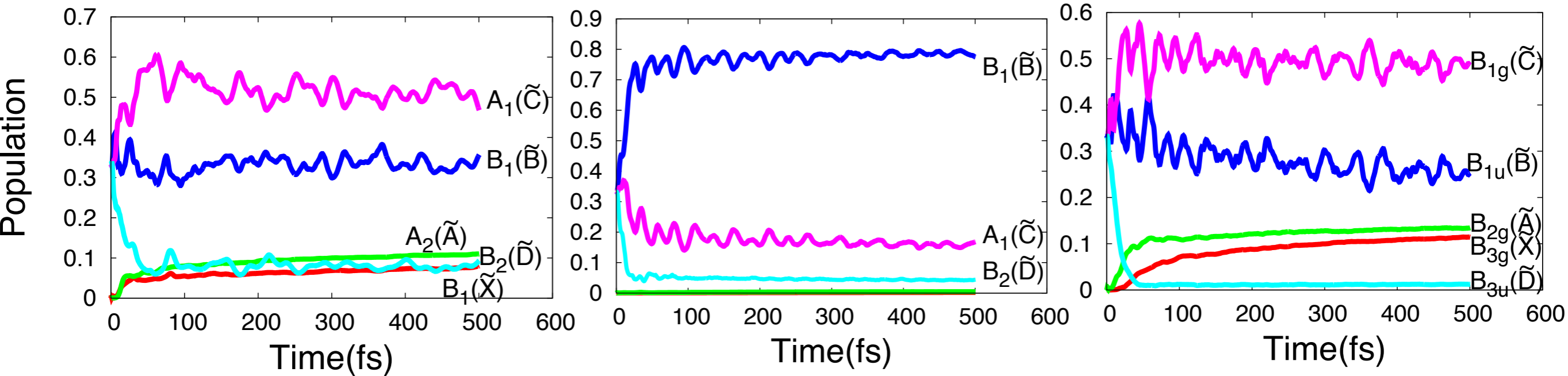
**Fluorescent**

**Non- fluorescent**

**Ortho**

**Meta**

**Para**



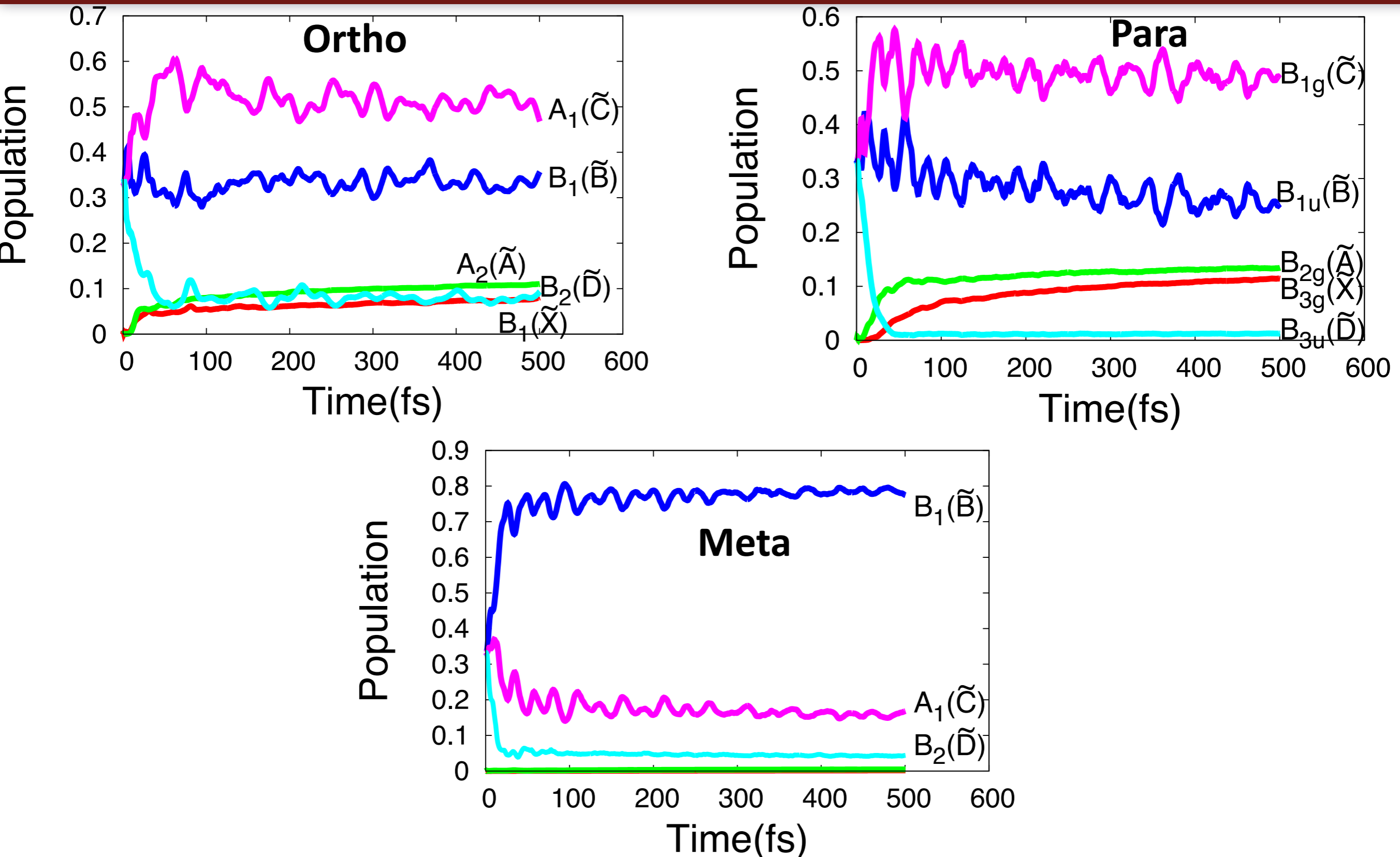
**No population transfer to the GS**

# Transition dipole moment

- Oscillator strengths of transition dipole moment between 5 electronic states

		$\tilde{\mathbf{B}}$	$\tilde{\mathbf{C}}$	$\tilde{\mathbf{D}}$
<i>Mono</i>	$\tilde{\mathbf{X}}$	0.0935	-	0.000
	$\tilde{\mathbf{A}}$	0.0552	0.000	-
1,2	$\tilde{\mathbf{X}}$	0.0999	0.0001	-
	$\tilde{\mathbf{A}}$	0.0644	-	0.0001
1,3	$\tilde{\mathbf{X}}$	0.1002	-	0.0002
	$\tilde{\mathbf{A}}$	0.0641	0.000	-
1,4	$\tilde{\mathbf{X}}$	0.1282	-	-
	$\tilde{\mathbf{A}}$	0.0474	-	0.000
1,2,3	$\tilde{\mathbf{X}}$	0.0889	-	0.000
	$\tilde{\mathbf{A}}$	0.0907	0.0002	-

# Time-dependent electronic populations



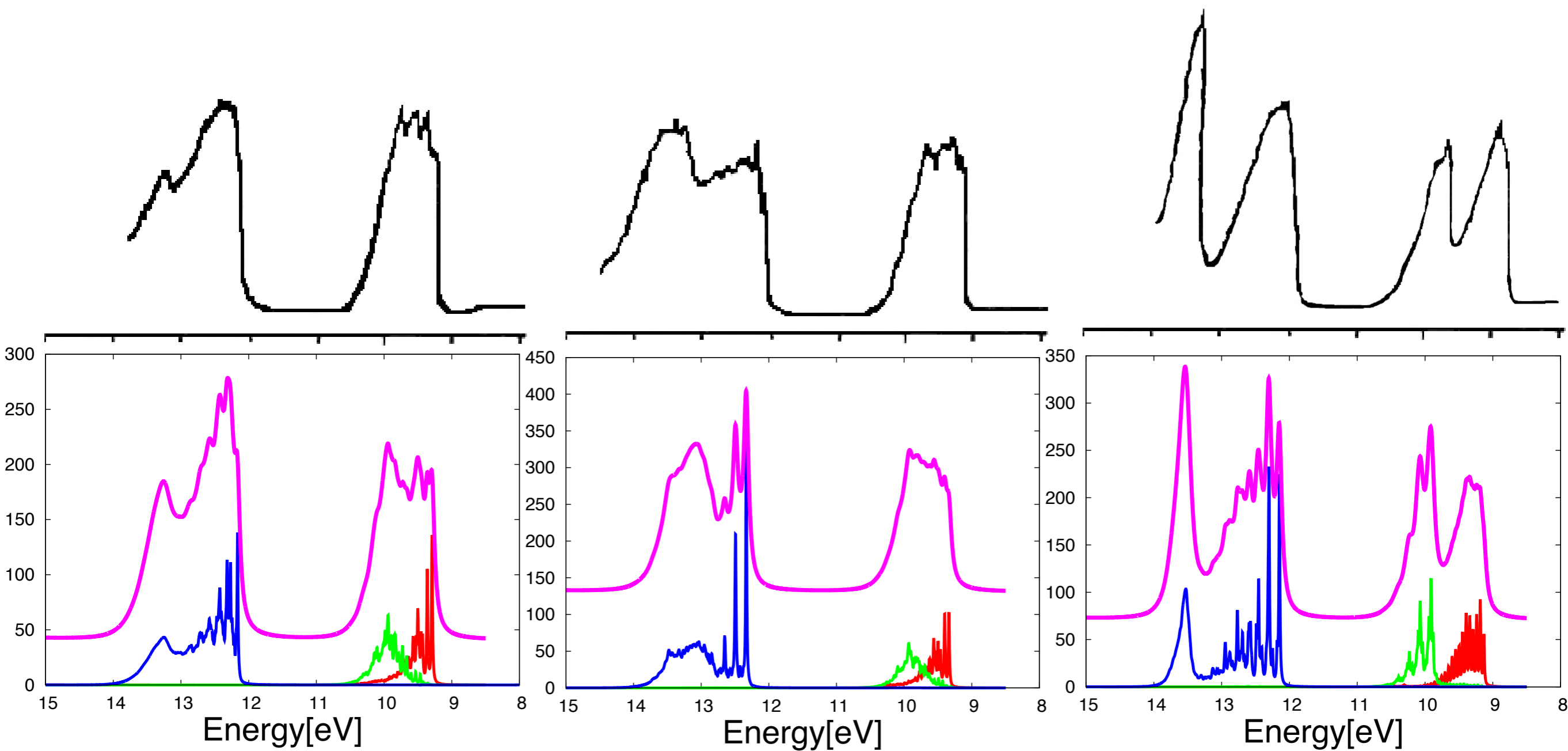


# Photoelectron spectra

Ortho

Meta

Para

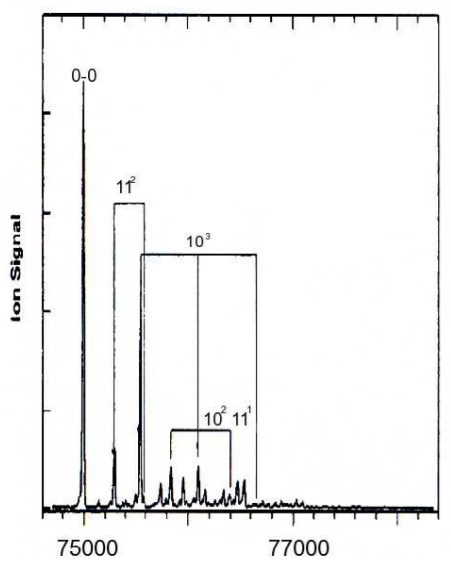




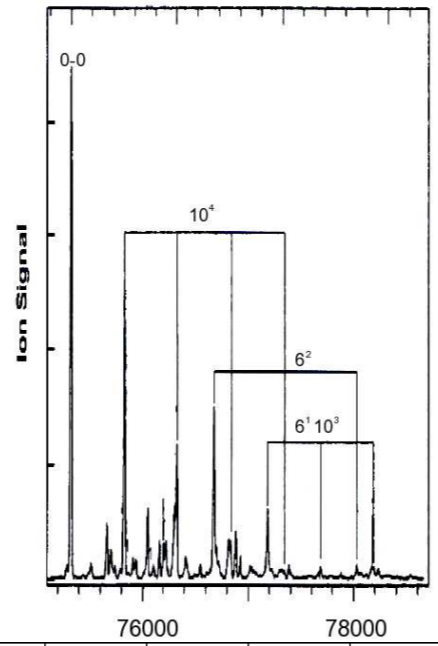


# MATI spectra

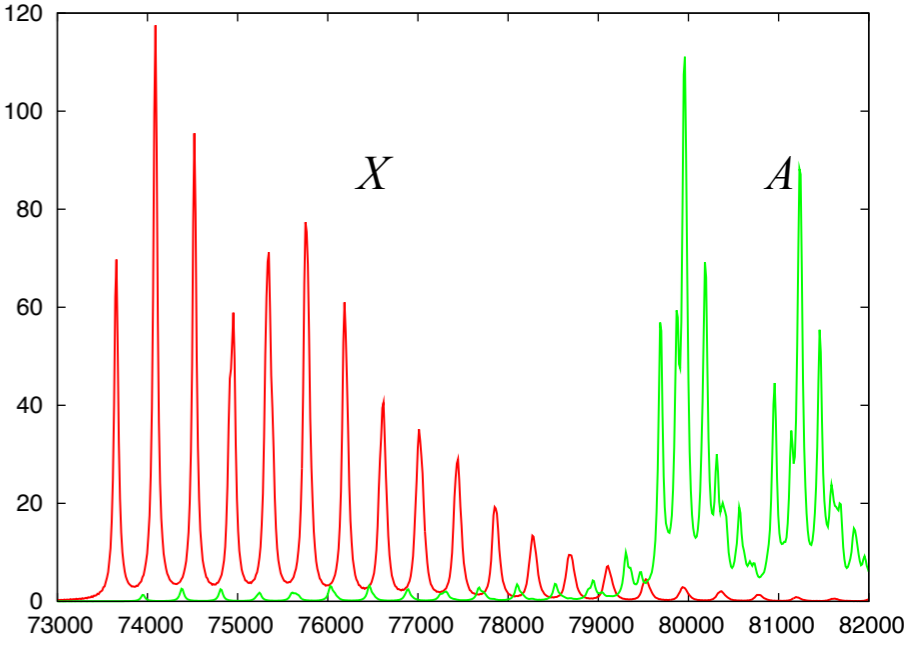
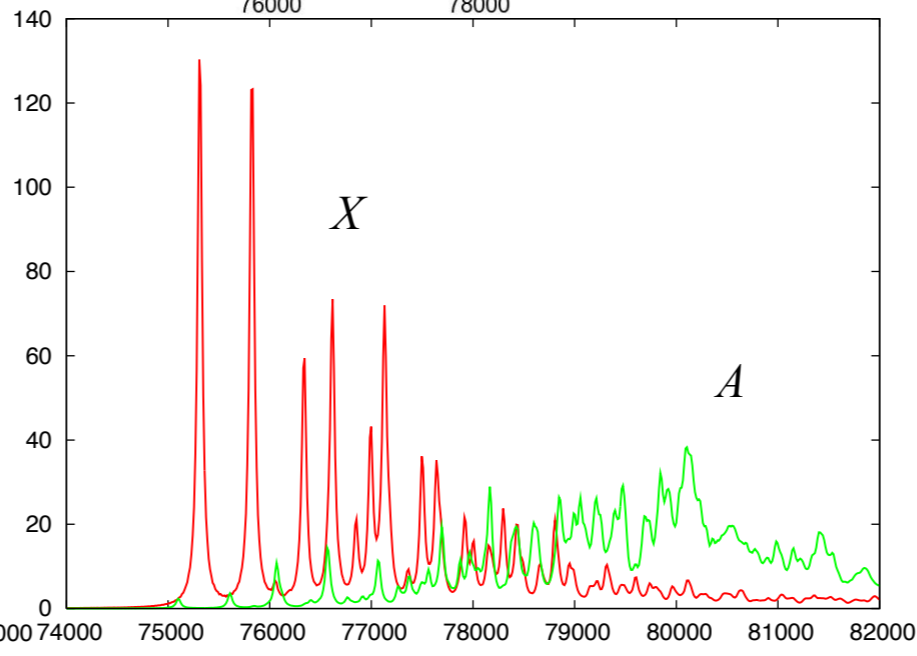
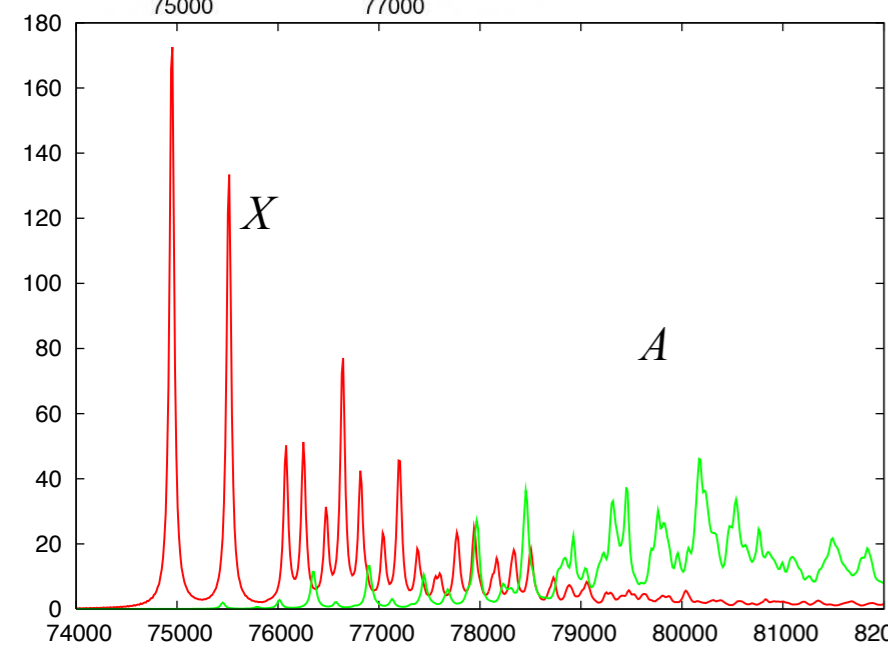
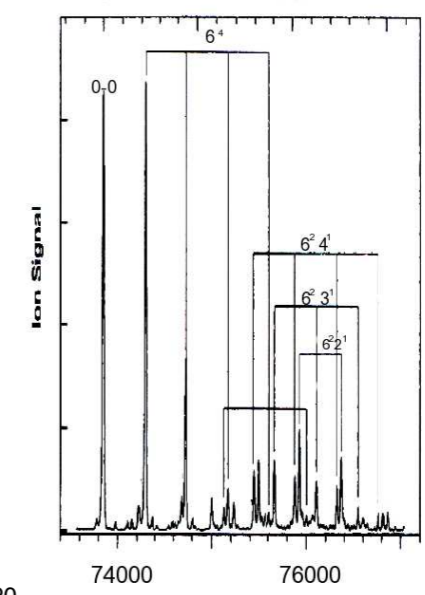
o-difluorobenzene



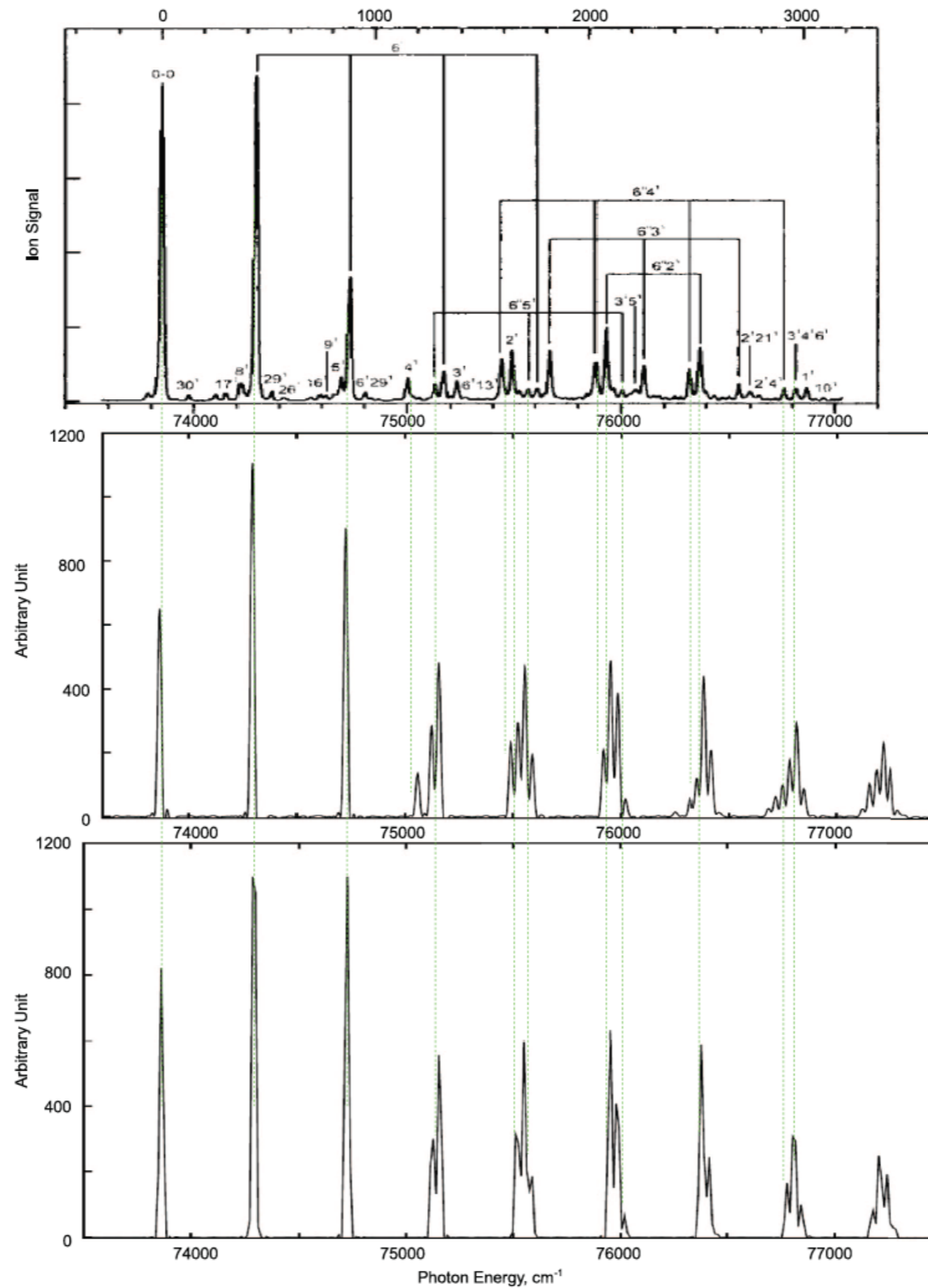
m-difluorobenzene



p-difluorobenzene

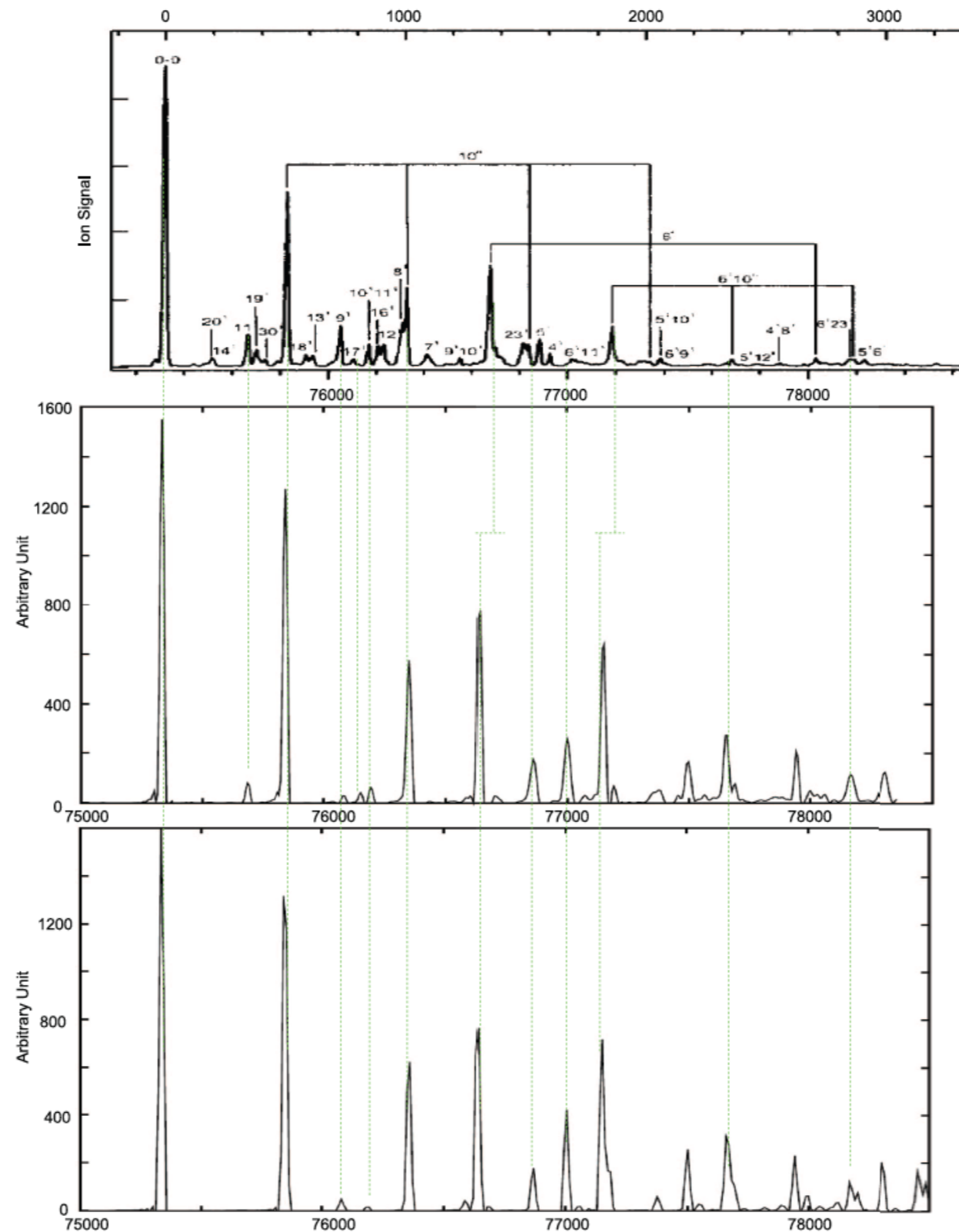


# Full dimension 30 DOF



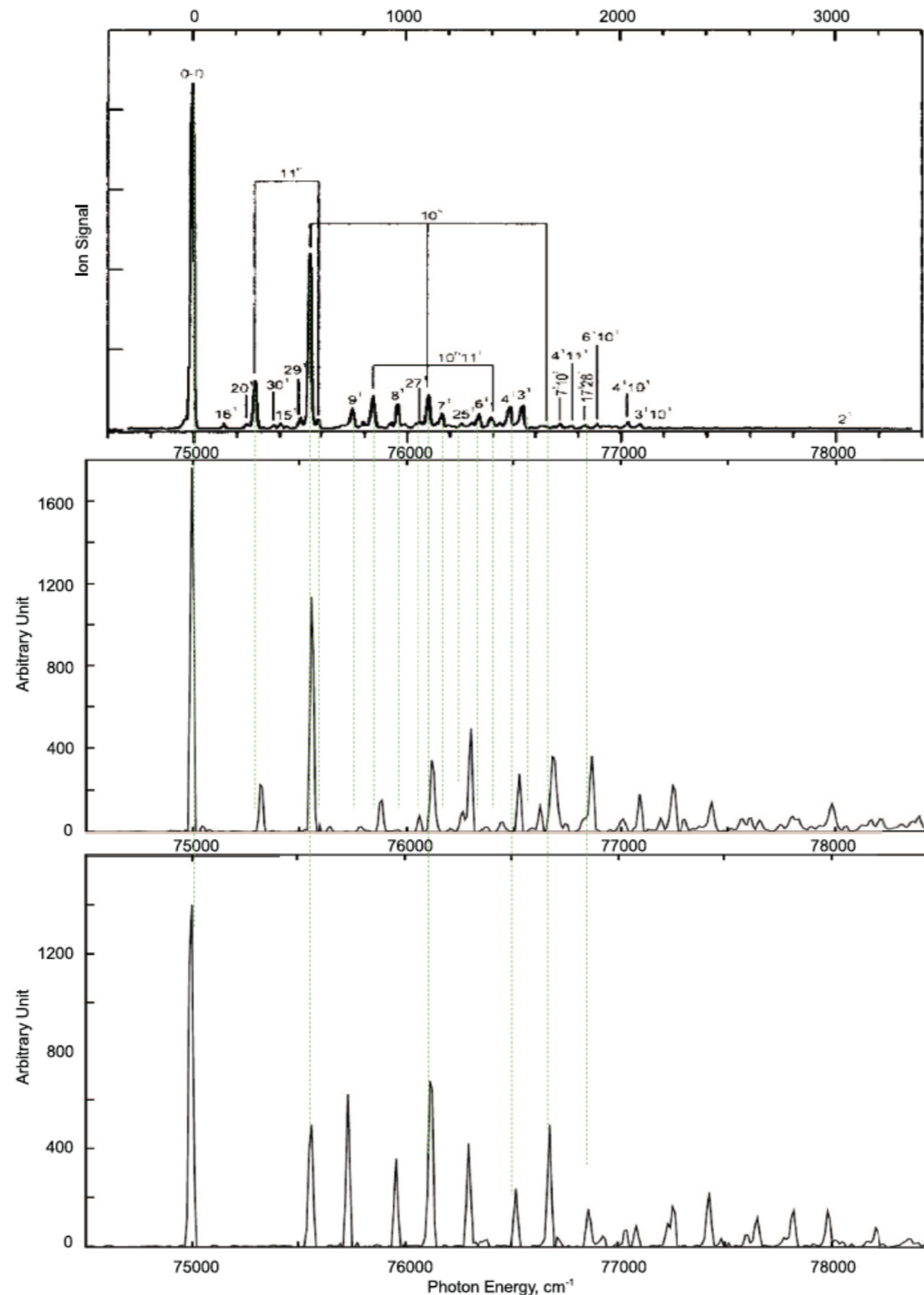


# Full dimension 30 DOF



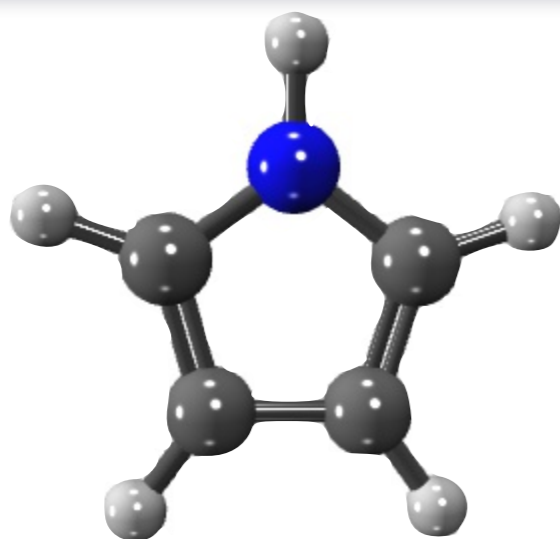


# Full dimension 30 DOF

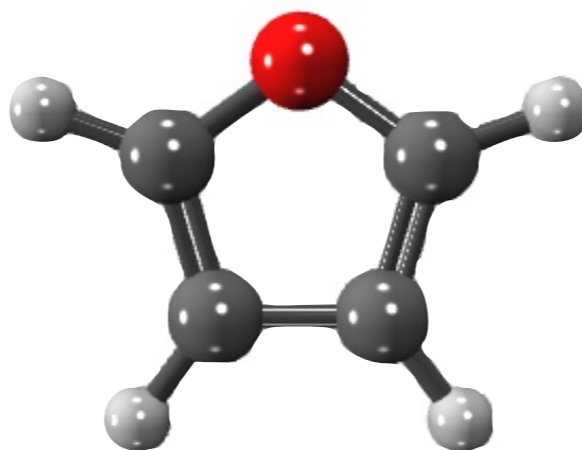




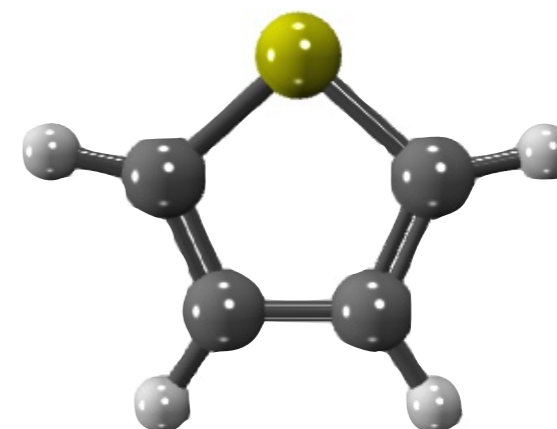
# Heteroaromatic molecules



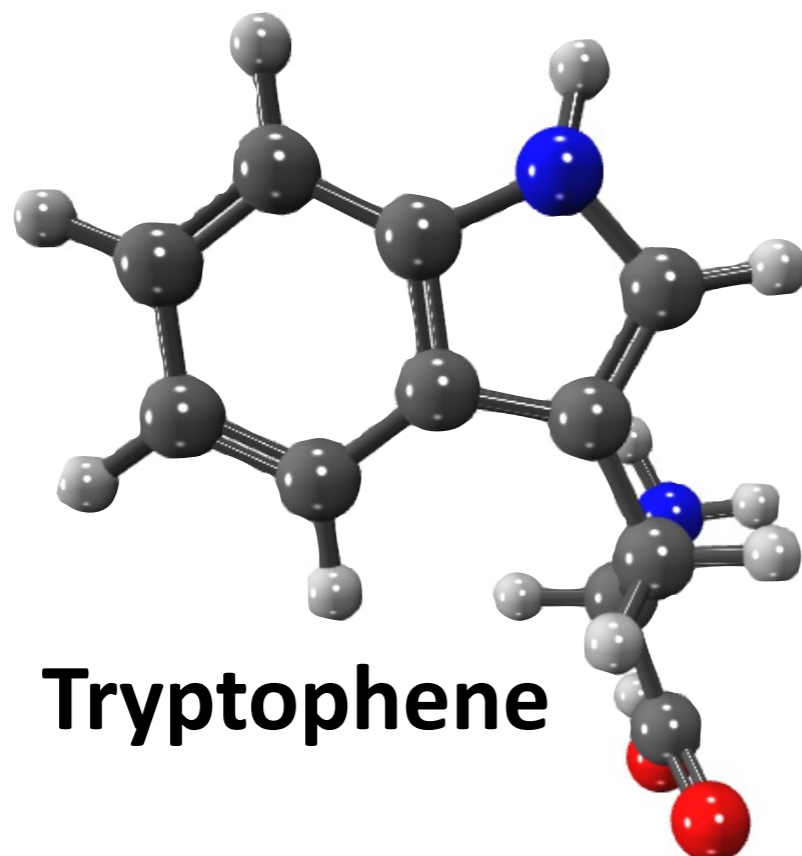
**Pyrrole**



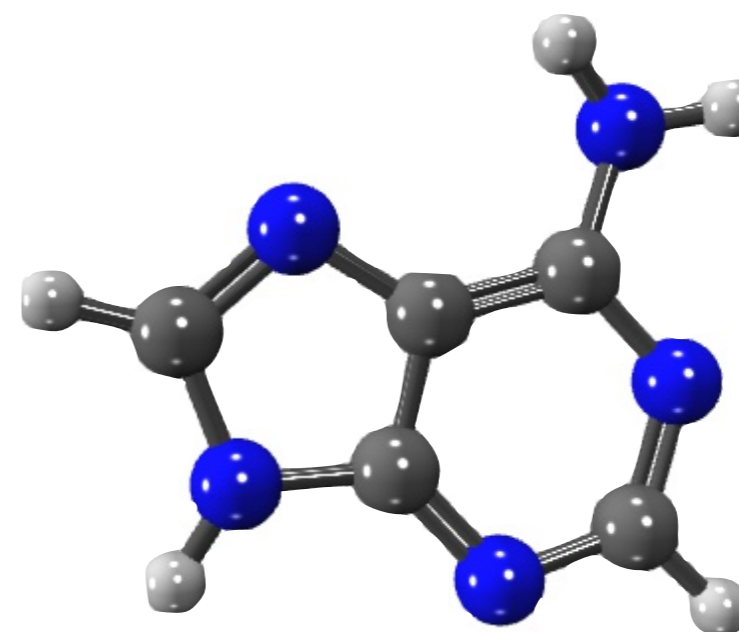
**Furan**



**Thiophene**



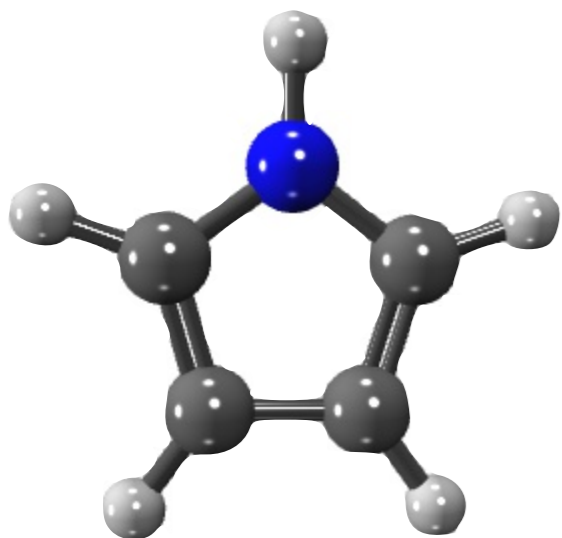
**Tryptophene**



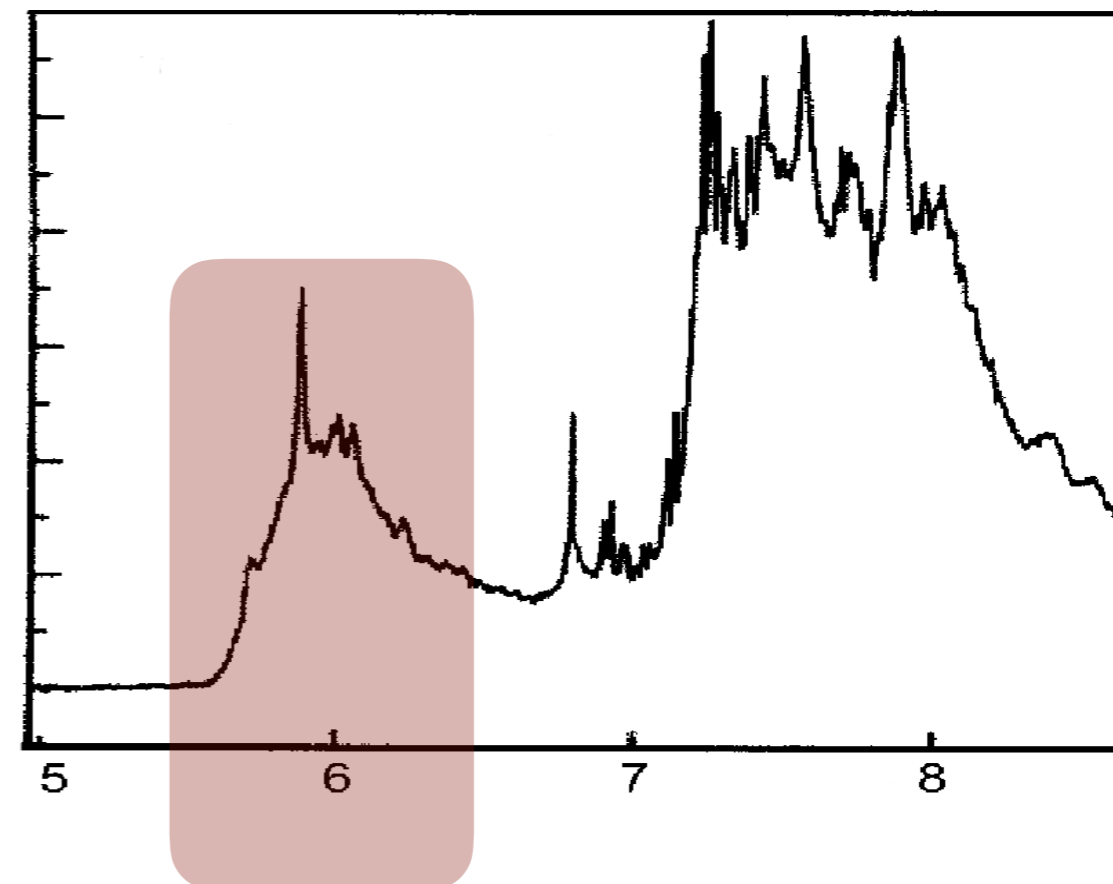
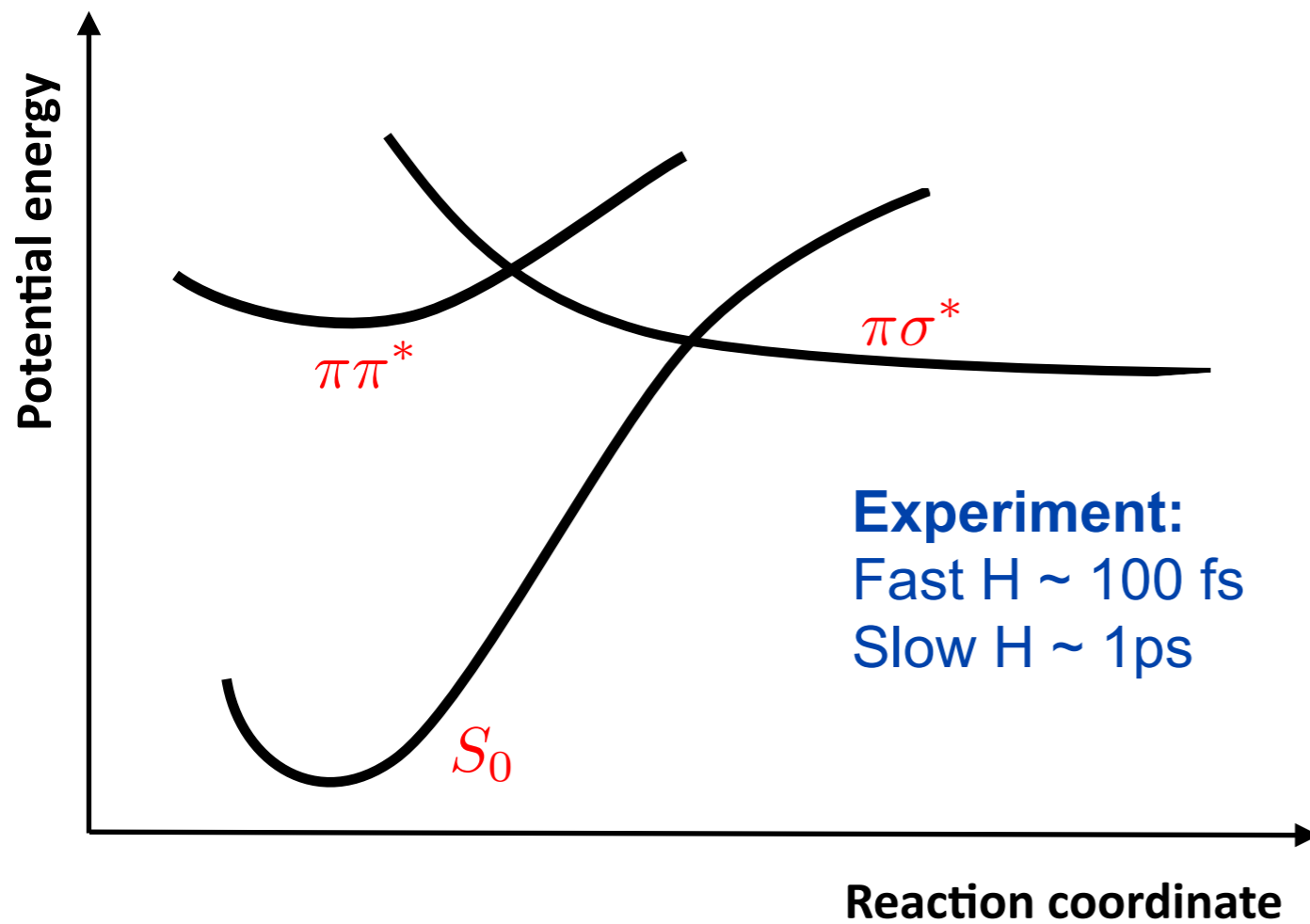
**Adenine**



# Pyrrole



$$\Gamma_{vib} = 9A_1 + 3A_2 + 4B_1 + 8B_2$$



A. L. Sobolewski and W. Domcke, Chem. Phys. 259, 181 (2000)

# 5 lowest electronic states

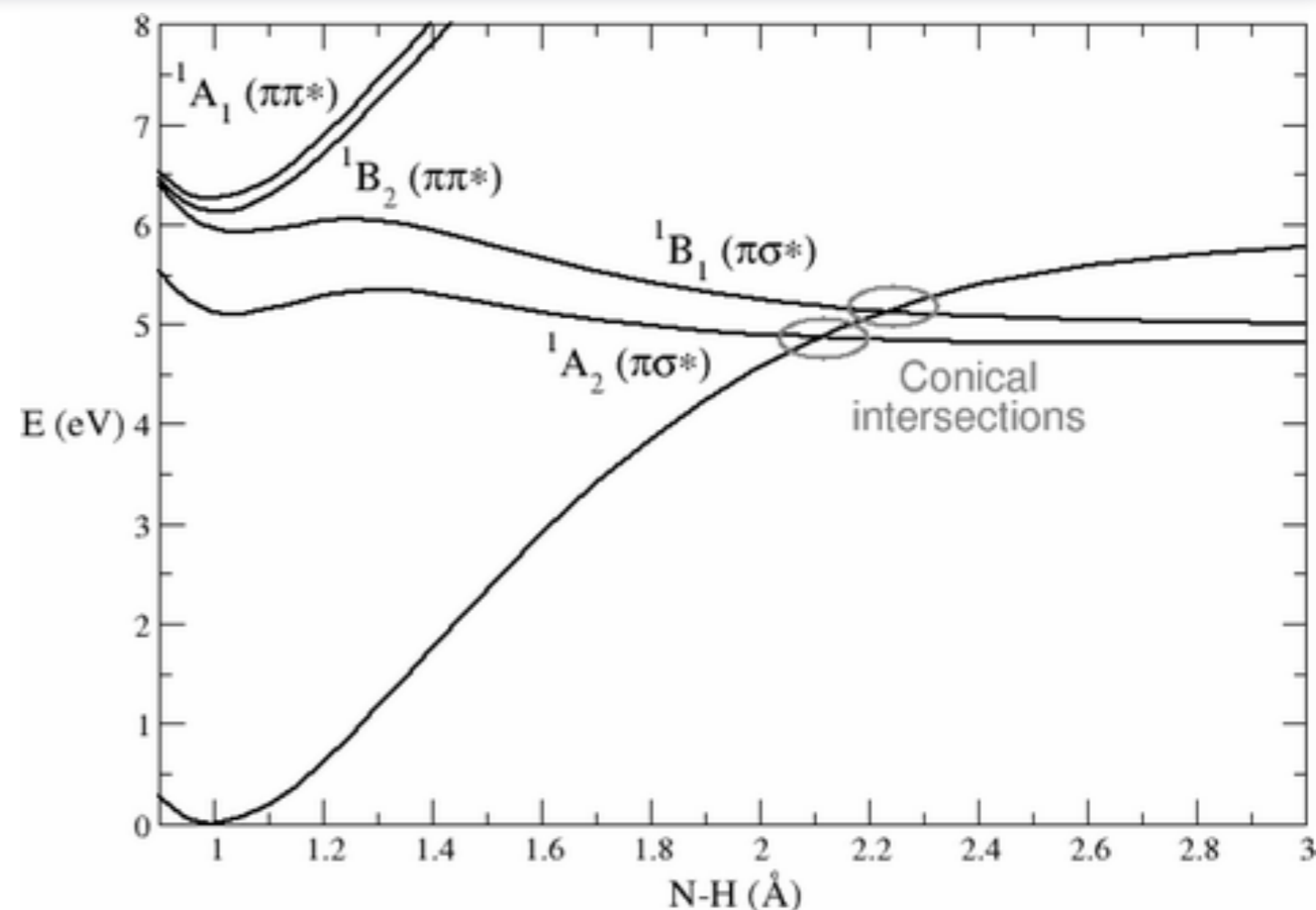
$B_2(\pi\pi^*)$  ————— 6.75 eV

$A_1(\pi\pi^*)$  ————— 6.55 eV

$B_1(\pi\sigma^*)$  ————— 6.12 eV

$A_2(\pi\sigma^*)$  ————— 5.33 eV

$A_1$  —————



A. L. Sobolewski and W. Domcke, Chem. Phys. 259, 181 (2000)

MR-CISD+Q/cc-pVDZ  
 CASSCF(5,6)

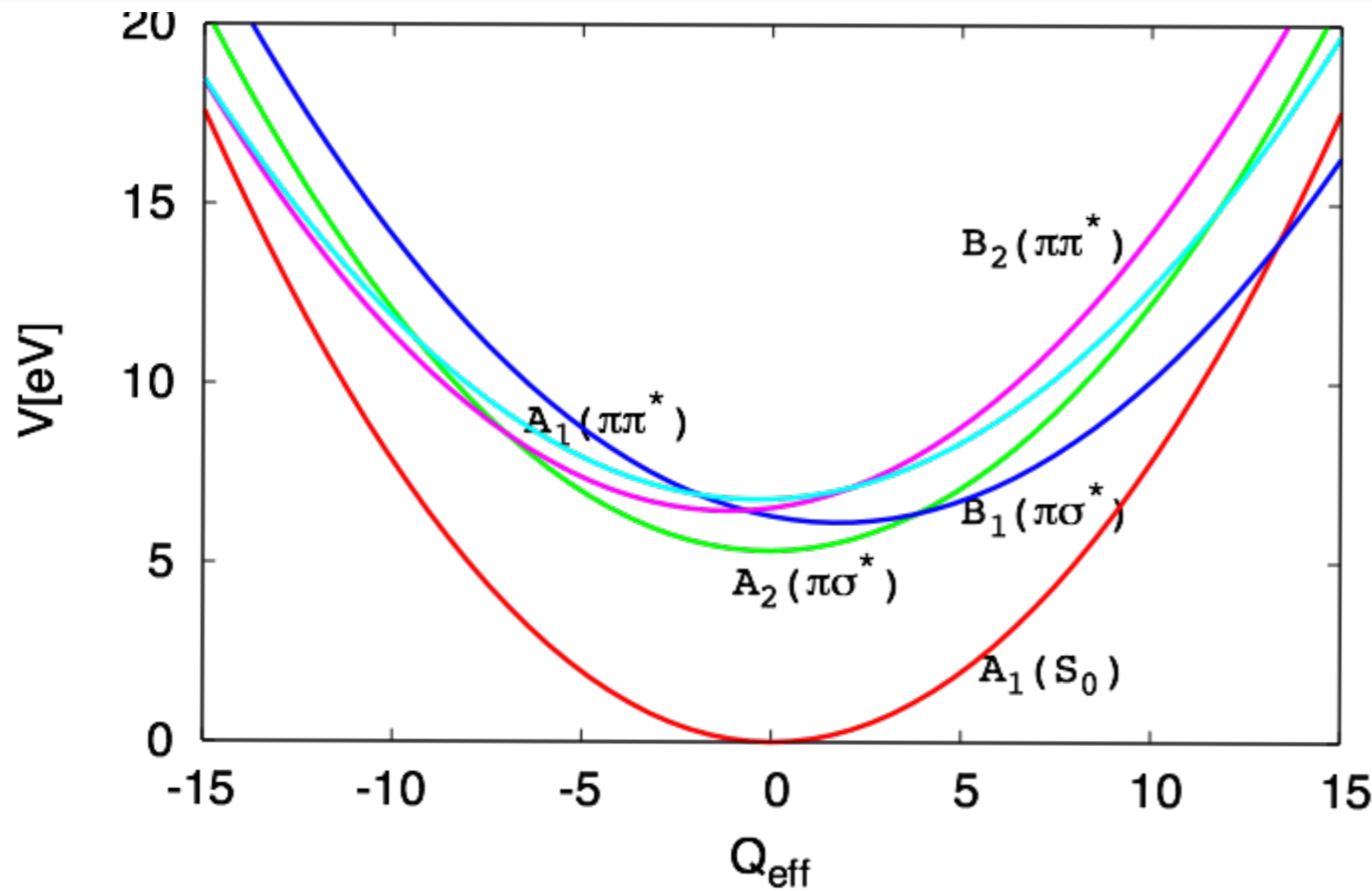
# 5x5 Vibronic Coupling Hamiltonian

$$\mathbf{H} = (T_N + V_0) \mathbf{1} + \mathbf{W}$$

$$\mathbf{W} = \begin{pmatrix} W_0 & \lambda_{4/7}^{(0,1)} Q_{4/7} & \lambda_6^{(0,2)} Q_6 & 0 & 0 \\ \lambda_{4/7}^{(0,1)} Q_{4/7} & W_1 & \lambda_{8,14,17}^{(1,2)} Q_{8,14,17} & \lambda_{4/7}^{(1,3)} Q_{4/7} & 0 \\ \lambda_6^{(0,2)} Q_6 & \lambda_{8,14,17}^{(1,2)} Q_{8,14,17} & W_2 & \lambda_3^{(2,3)} Q_3 & \lambda_{4/7}^{(2,4)} Q_{4/7} \\ 0 & \lambda_{4/7}^{(1,3)} Q_{4/7} & \lambda_3^{(2,3)} Q_3 & W_3 & \lambda_{8,14,17}^{(3,4)} Q_{8,14,17} \\ 0 & 0 & \lambda_{4/7}^{(2,4)} Q_{4/7} & \lambda_{8,14,17}^{(3,4)} Q_{8,14,17} & W_4 \end{pmatrix}$$

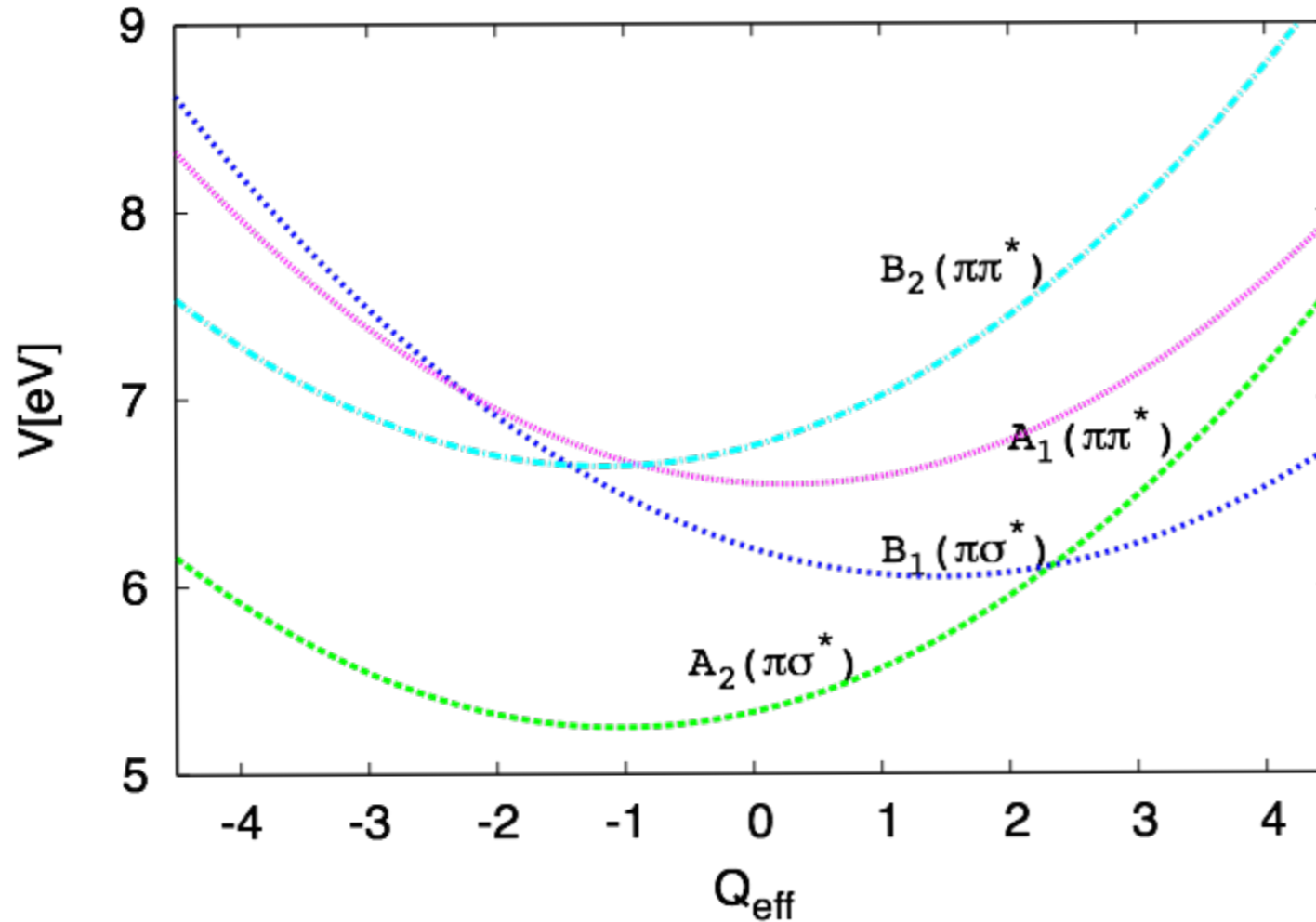


# Minima of conical intersection seams S0-S4



	$S_0$	$S_1$	$S_2$	$S_3$	$S_4$
$S_0(A_1)$	0.0	13.82	14.06	13.05	14.44
$S_1(A_2)$		5.01	6.04	6.49	9.28
$S_2(B_1)$			6.01	6.29	6.58
$S_3(A_1)$				6.25	6.57
$S_4(B_2)$					6.56

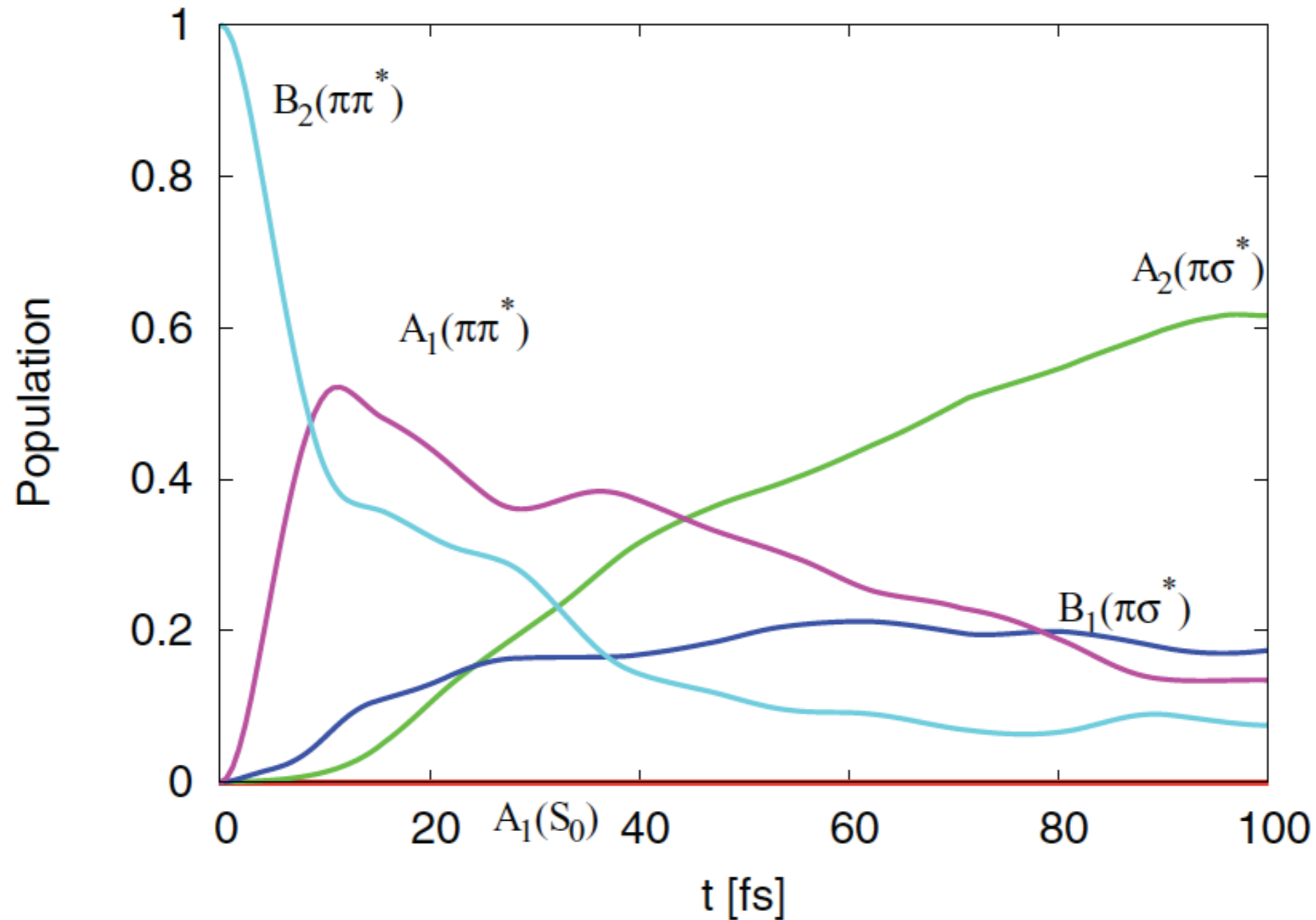
# Minima of conical intersection seams S1-S2



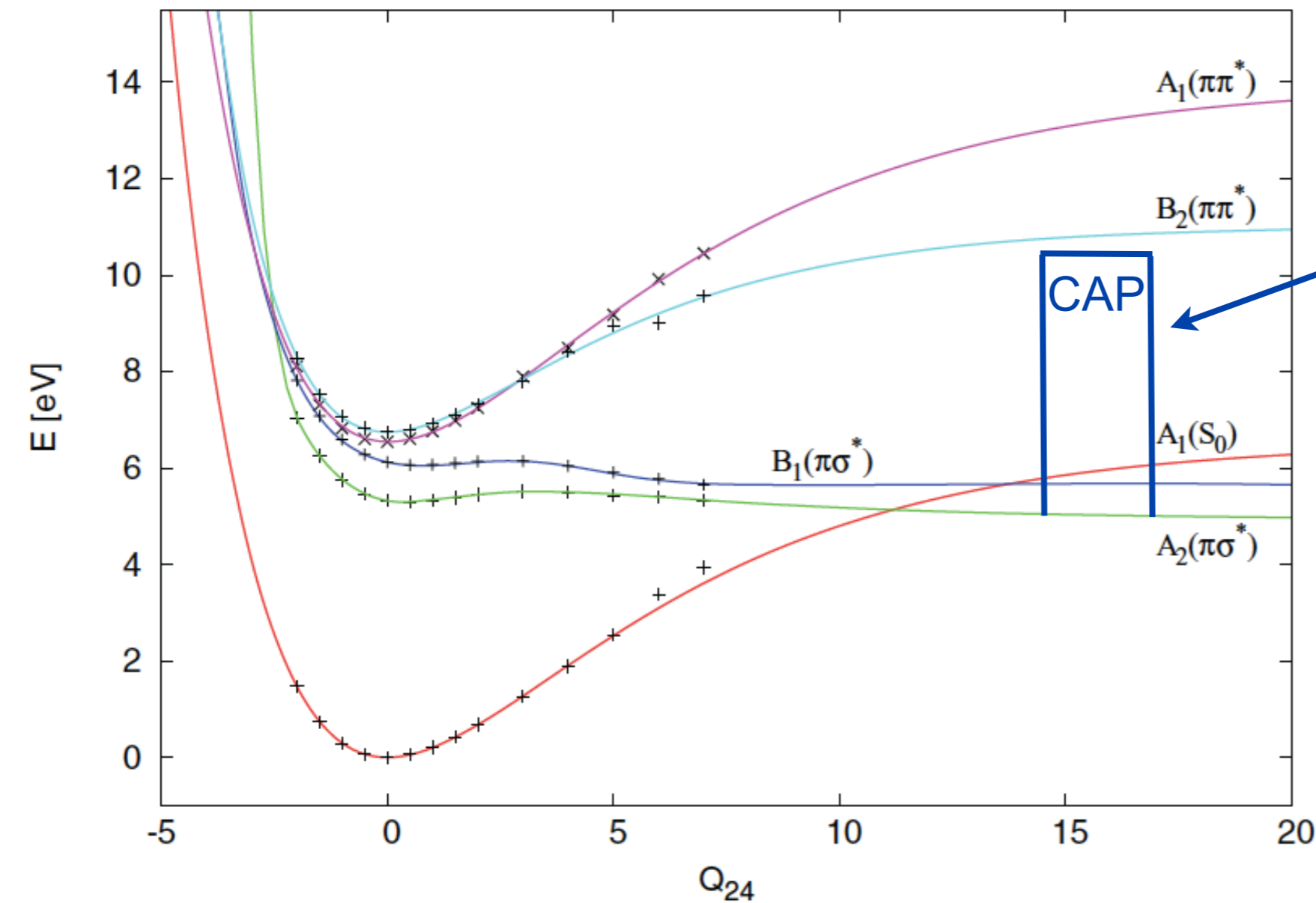
	$S_1$	$S_2$	$S_3$	$S_4$
$S_1$	5.05	6.00	6.71	8.59
$S_2$		5.95	6.29	6.48
$S_3$			6.28	6.48
$S_4$				6.48

# Time-dependent electronic populations

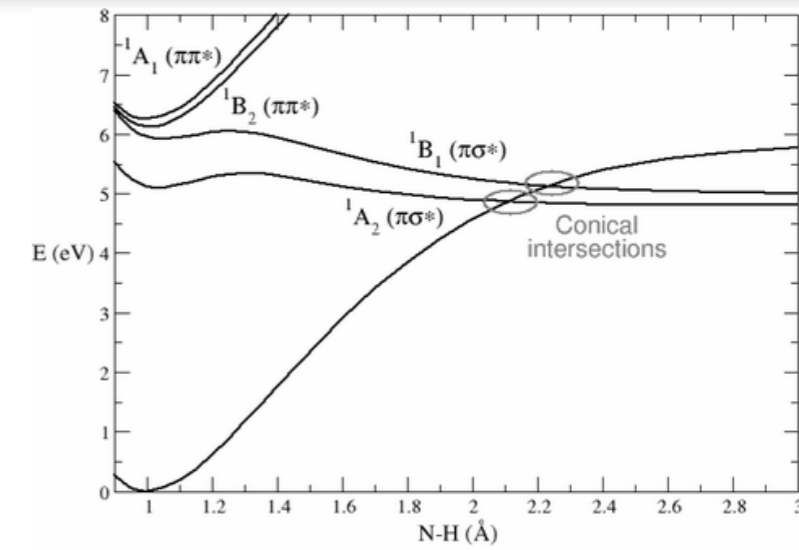
## 10 Mode, 5 States



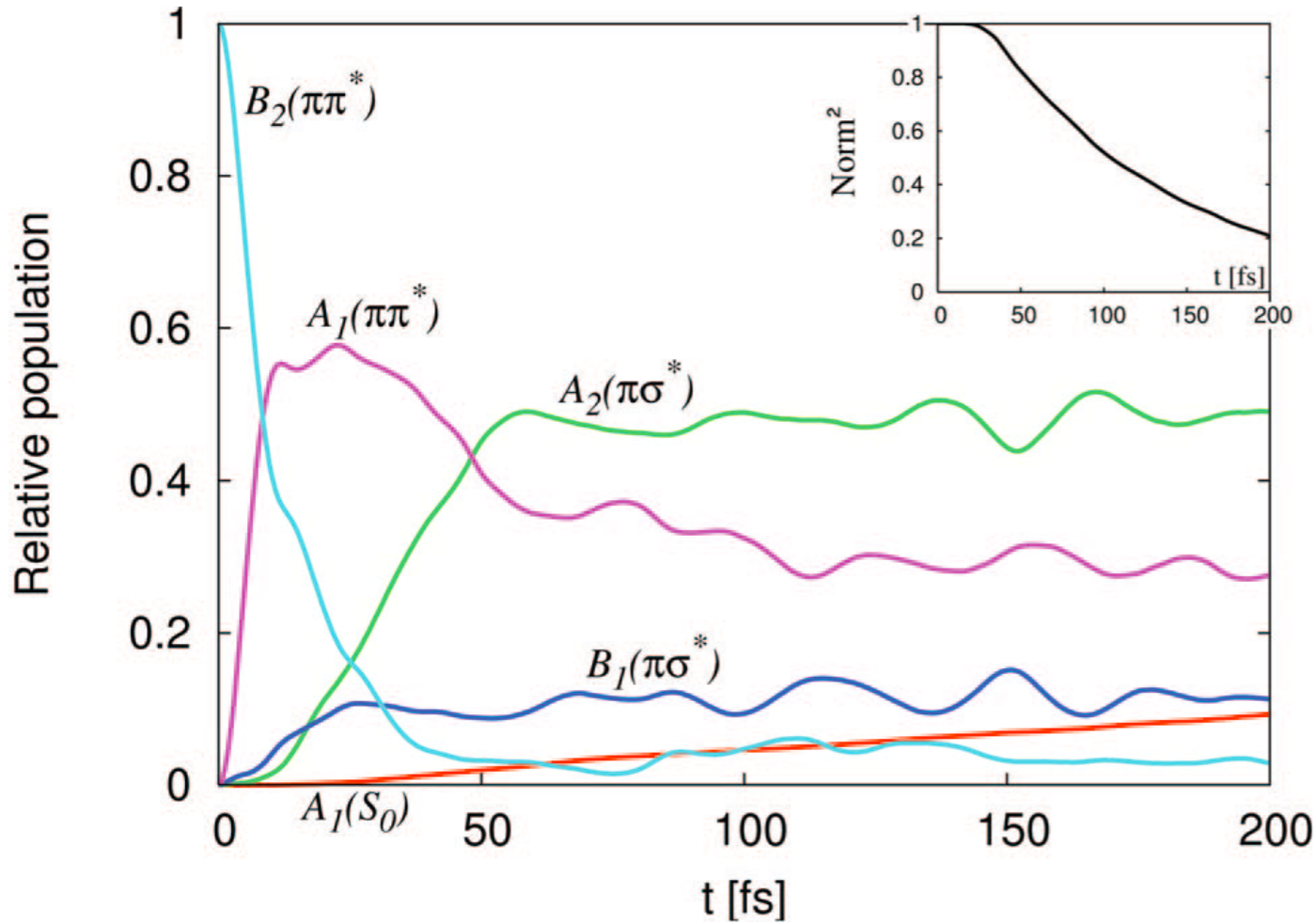
# Anharmonic treatment of mode 24



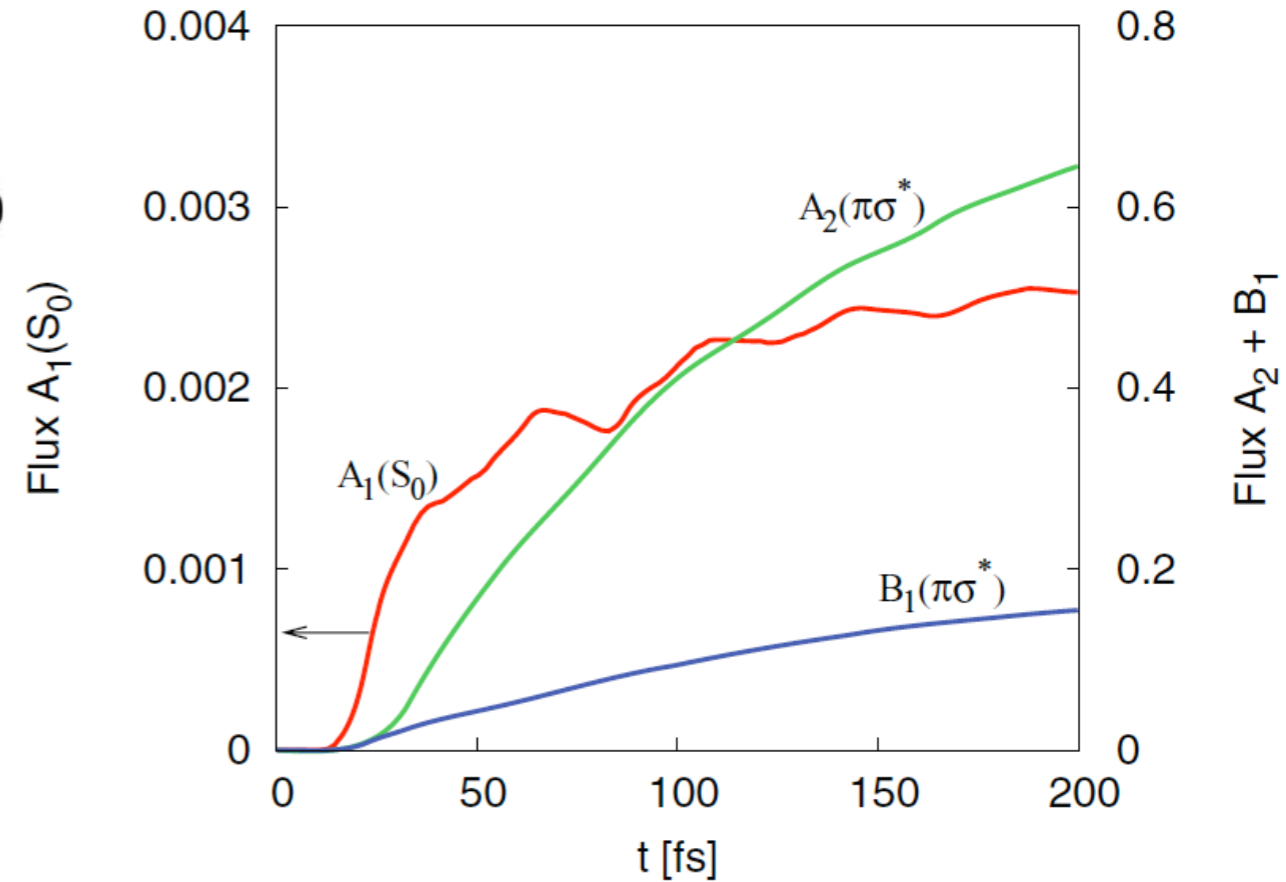
Complex  
 Absorbing  
 Potential



# Time-dependent electronic populations



Population transfer to  $S_0$



**Experiment:**  
 Fast H  $\sim 100$  fs  
 Slow H  $\sim 1$  ps

# Absorption spectrum

G. Bieri, L. Asbrink, and W. von Niessen, J. Electron Spectrosc. Relat. Phenom. **23**, 281 (1981)

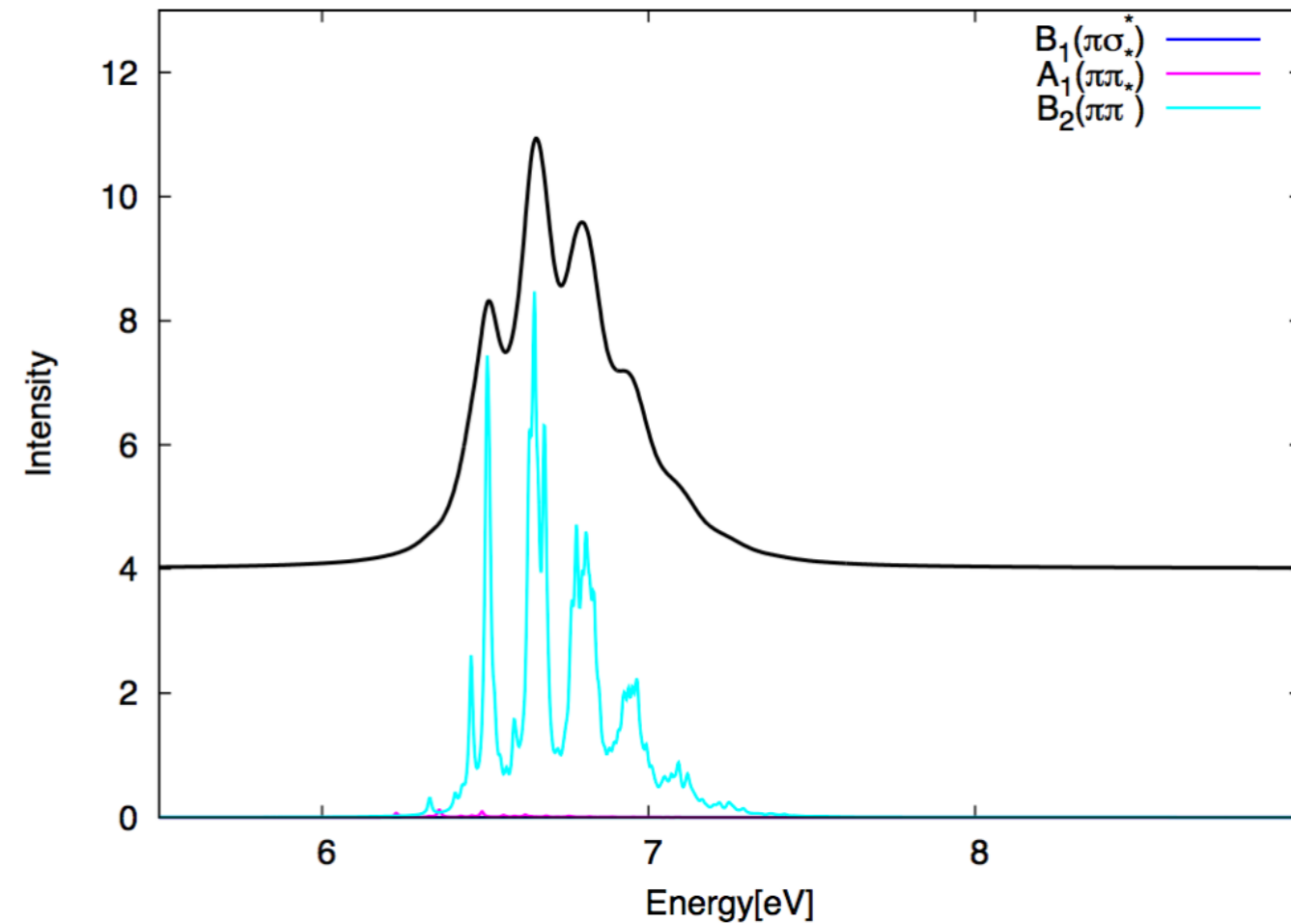
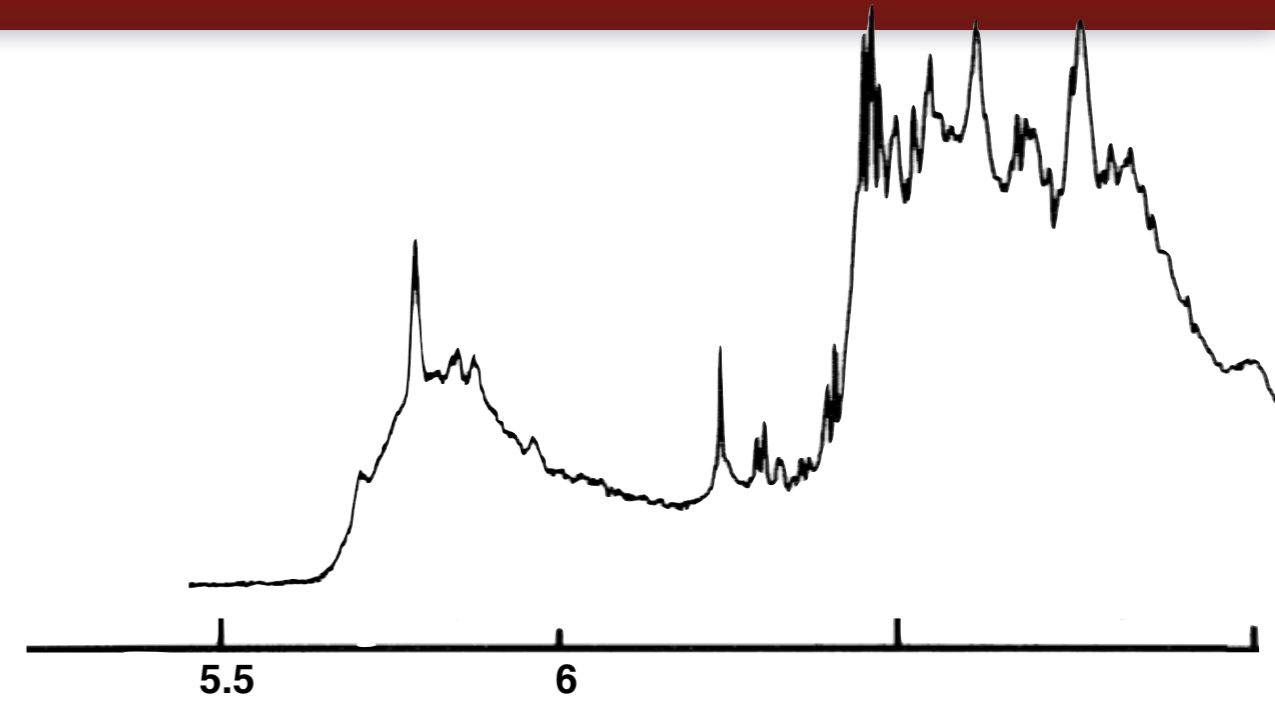
$B_2(\pi\pi^*)$  ————— 6.75

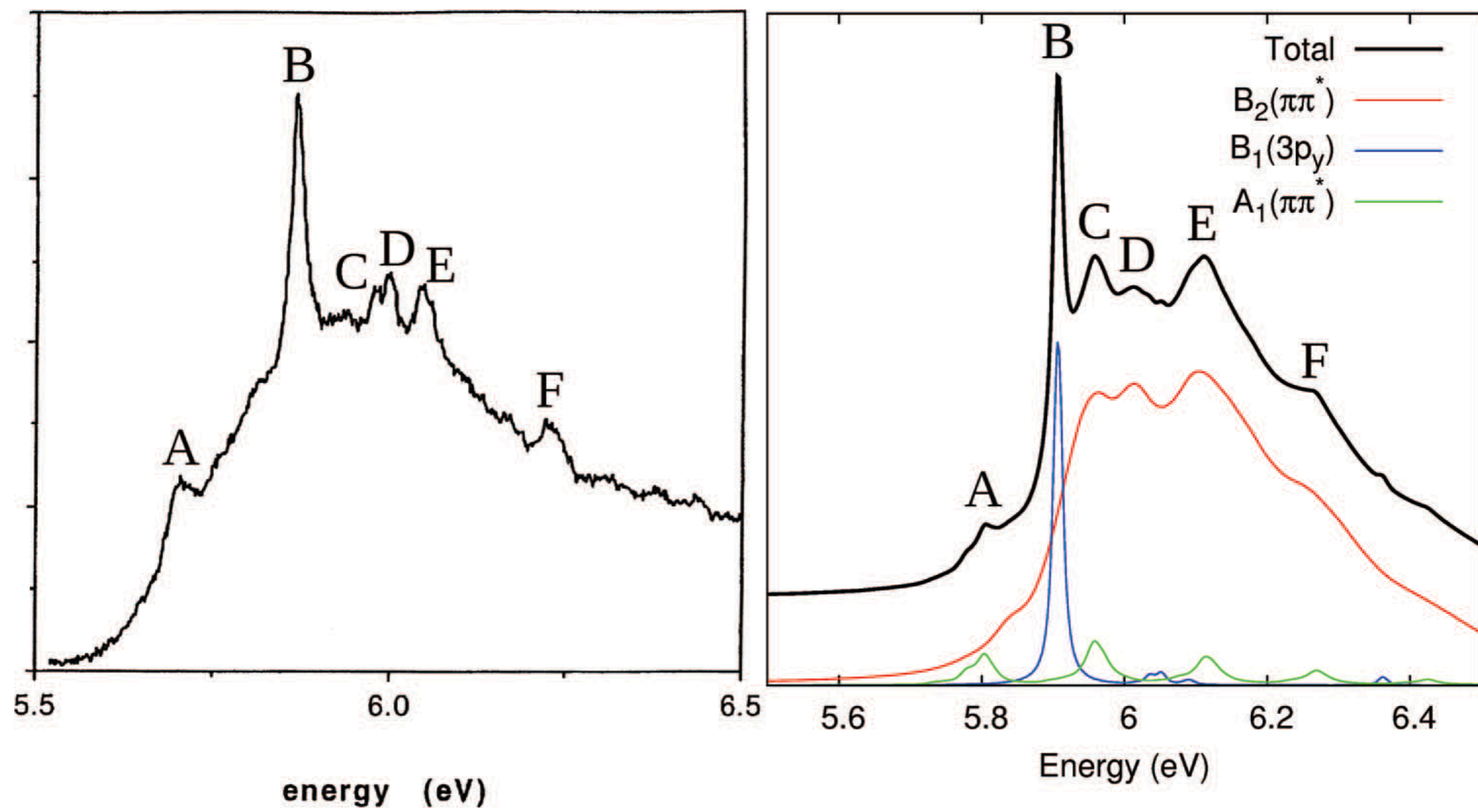
$A_1(\pi\pi^*)$  ————— 6.55

$B_1(\pi\sigma^*)$  ————— 6.12

$A_2(\pi\sigma^*)$  ————— 5.33

$A_1$  —————





S. P. Neville and G. A. Worth, J. Chem. Phys. 140, 034317 (2014).



MRCI+Q

CASSCF & EOM-CCSD

EOM-CCSD

$B_2(\pi\pi^*)$



6.75 eV

6.24 eV

6.26 eV

$A_1(\pi\pi^*)$



6.55 eV

6.01 eV

6.15 eV

$B_1(3P_y)$



6.00 eV

6.00 eV

$A_2(3P_z)$



5.87 eV

5.91 eV

$B_1(\pi\sigma^*)$



6.12 eV

5.86 eV

5.87 eV

$A_2(\pi\sigma^*)$



5.33 eV

5.06 eV

5.14 eV

$A_1$



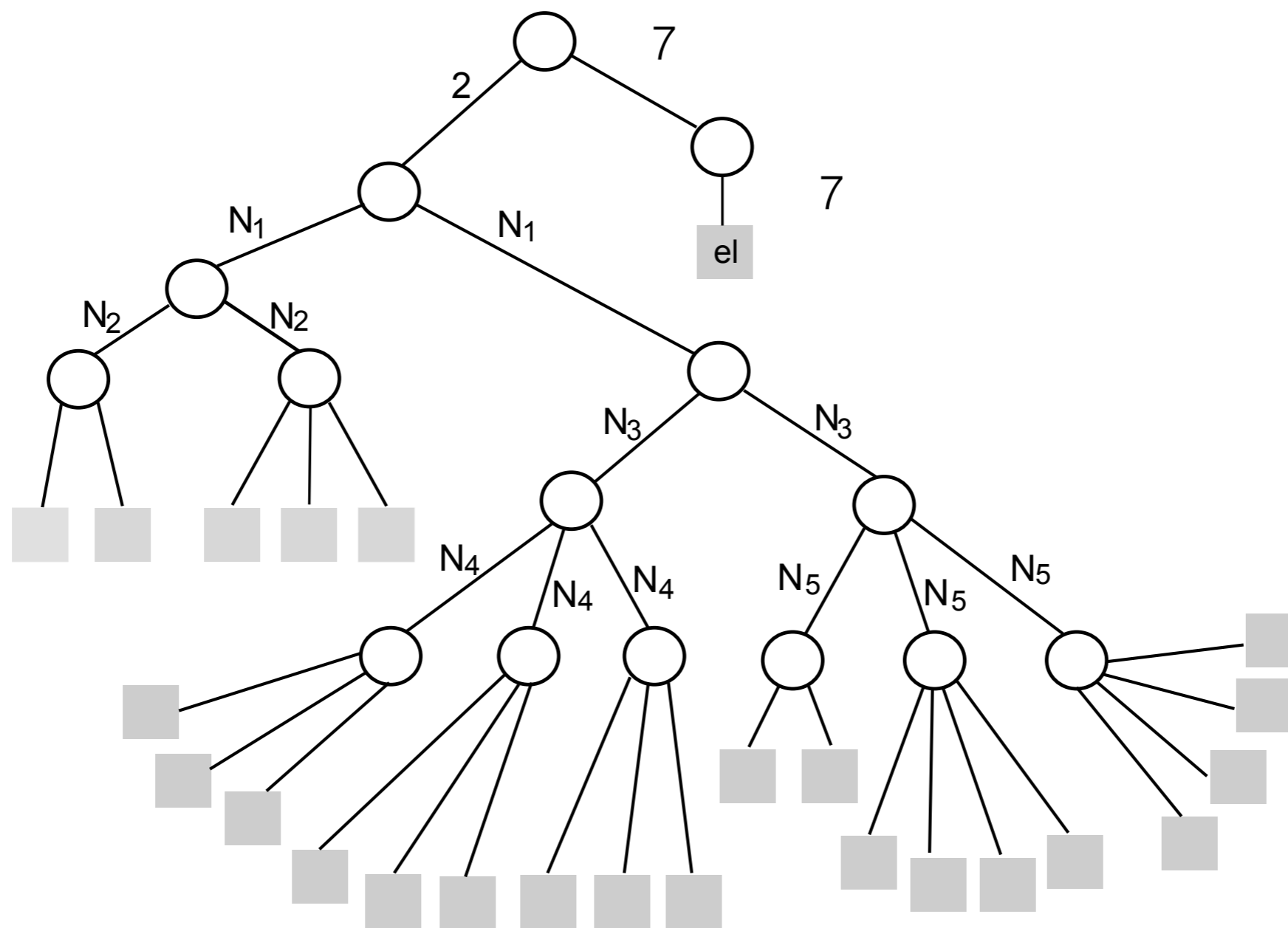


# Re-visit LVC(+Q) model Hamiltonian

- 7x7 model Hamiltonian ( $S_0, S_1-S_6$ )
- Same anharmonic treatment of N-H normal coordinate
- Same CAP
- 24 vibrational mode (full dimension)



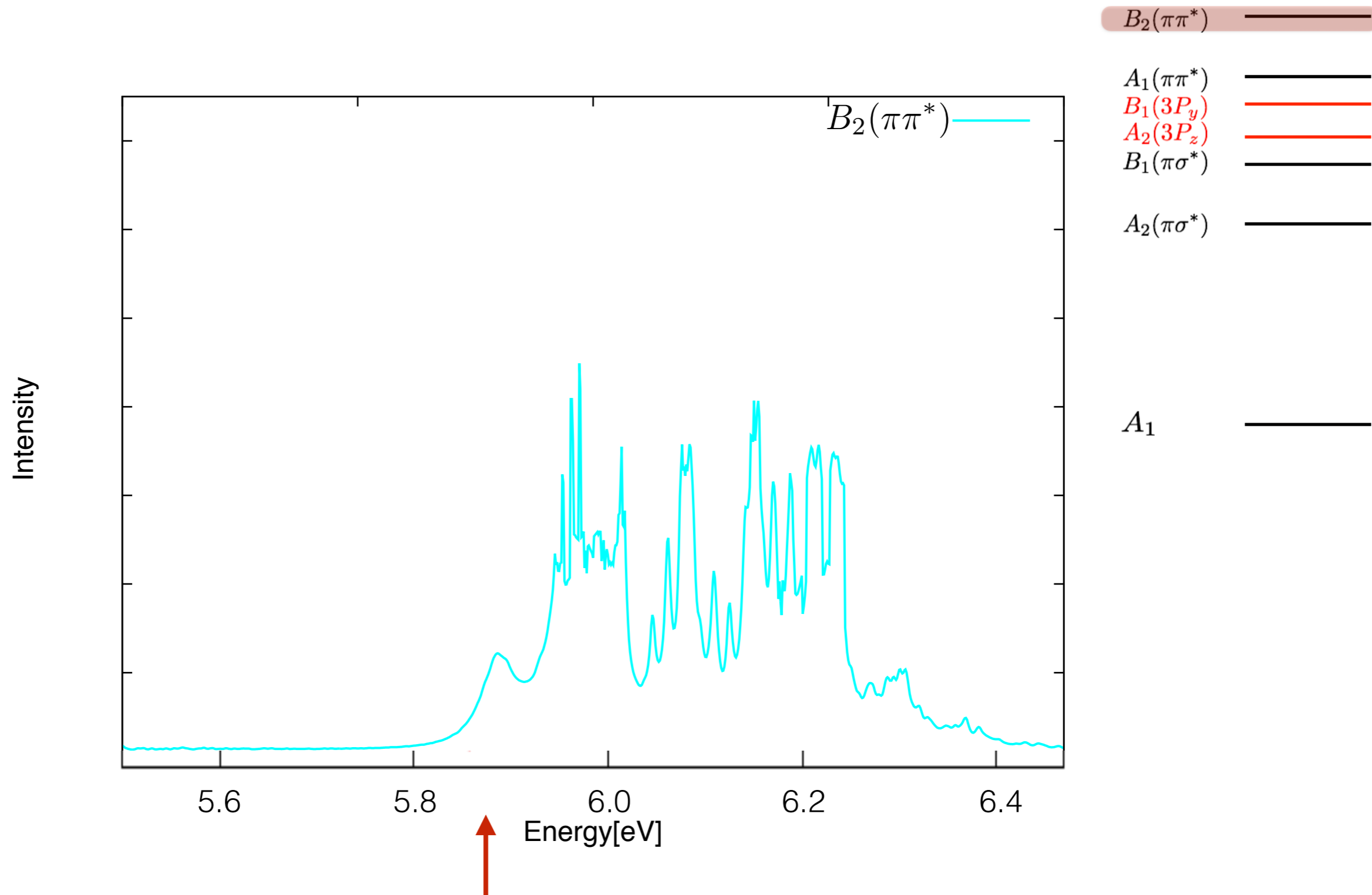
# Pyrrole 24D, ML-MCTDH-tree



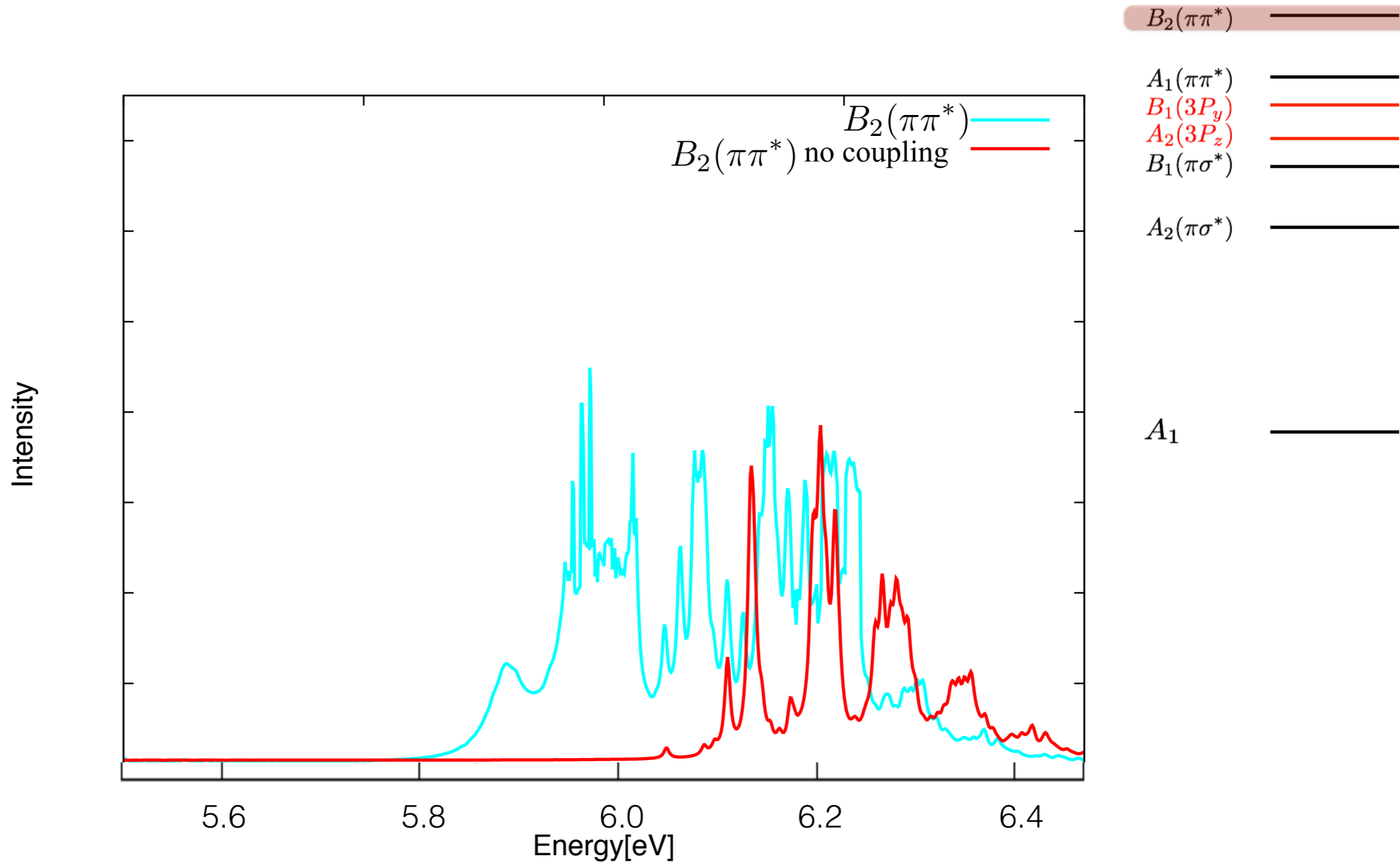
MCTDH (289 h)

ML-MCTDH (11.5 h)

# Absorption spectrum

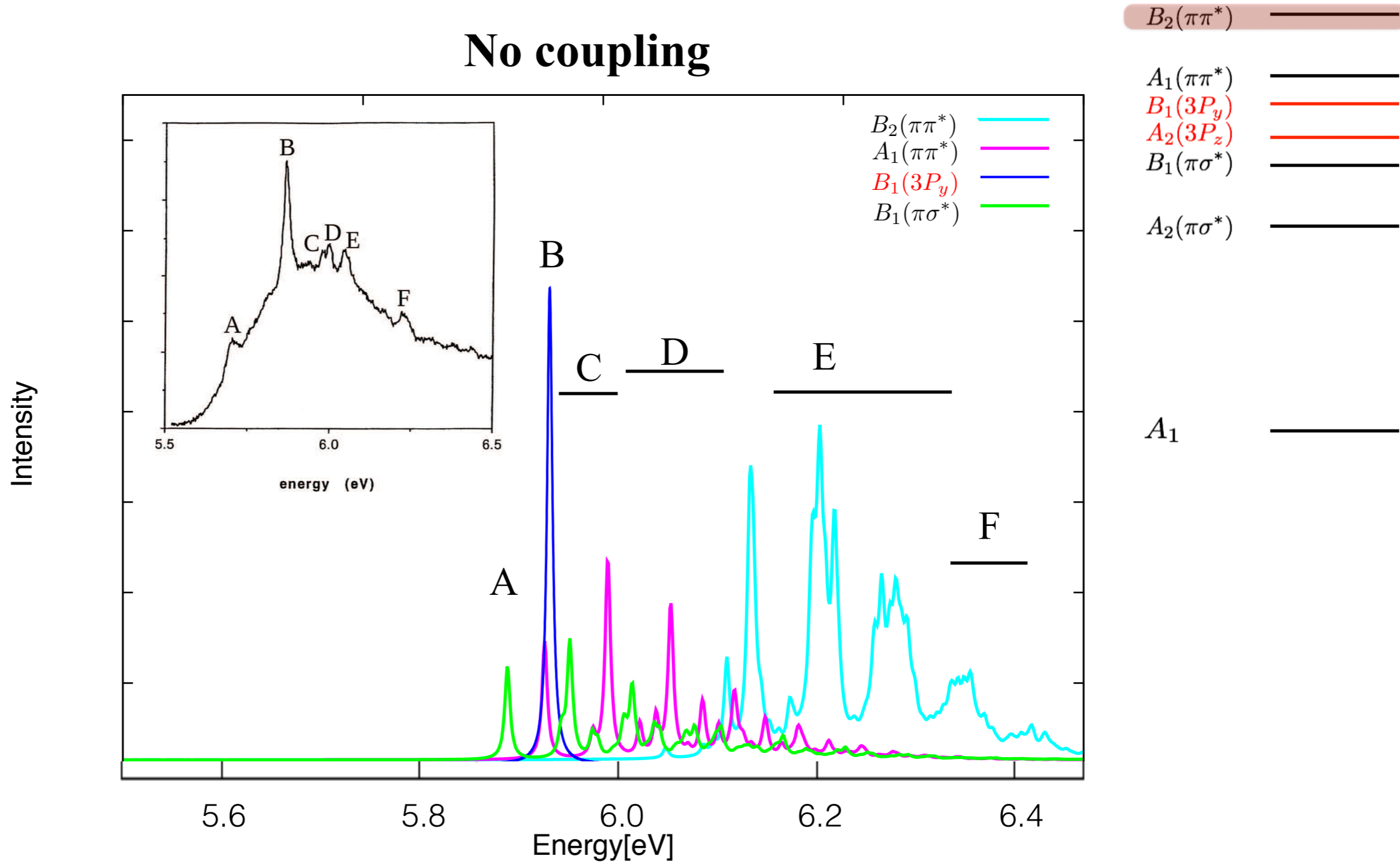


# Absorption spectrum



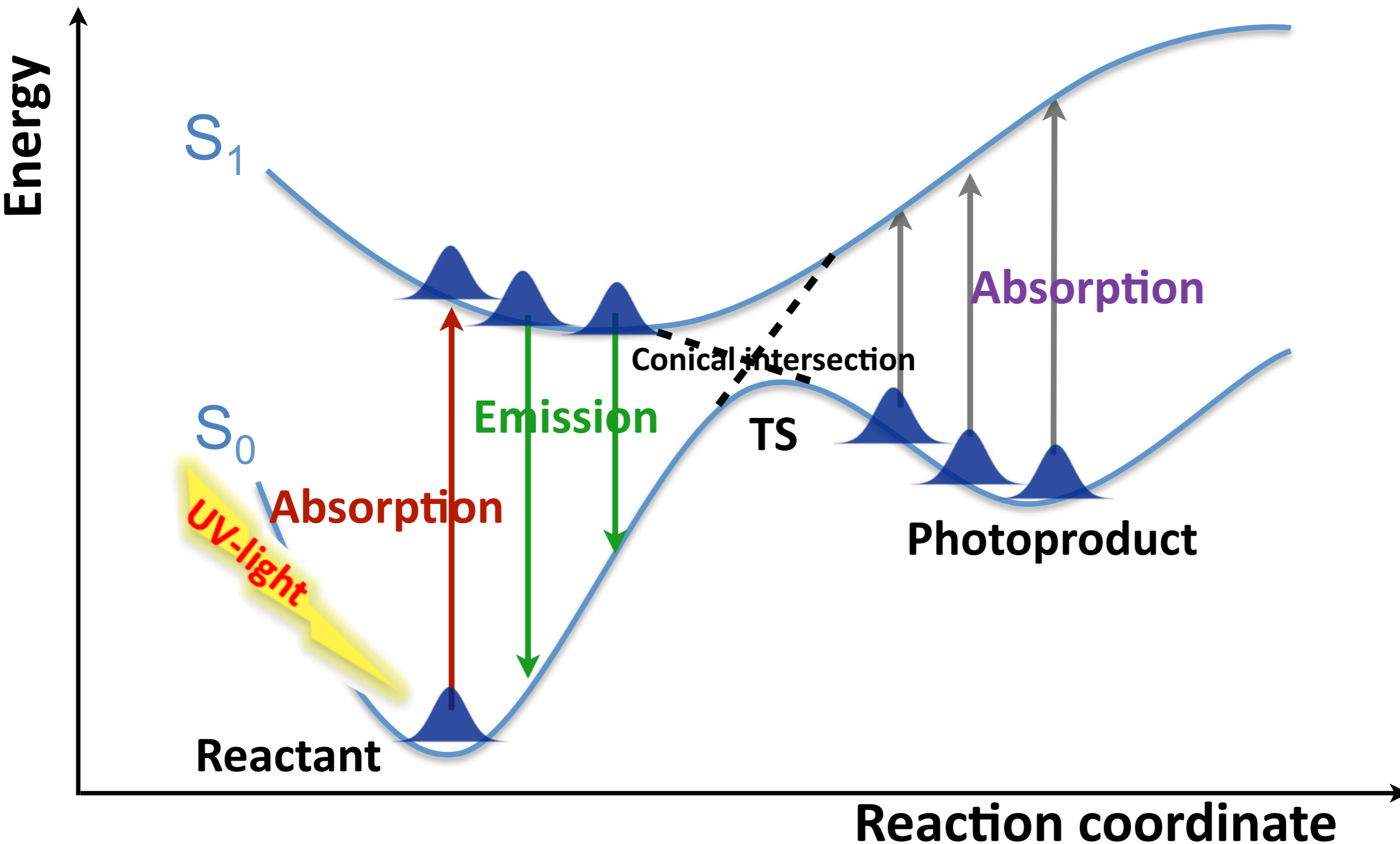
# Absorption spectrum

No coupling





# Theoretical challenges in quantum dynamics



# Gaussian based MCTDH

## ■ MCTDH

$$\Psi(Q_1, \dots, Q_f, t) = \sum_{j_1}^{n_1} \dots \sum_{j_f}^{n_f} A_{j_1 \dots j_f}(t) \prod_{\kappa=1}^f \varphi_{j_\kappa}^{(\kappa)}(Q_\kappa, t)$$

## ■ Gaussian based MCTDH (G-MCTDH)

$$\Psi(Q_1, \dots, Q_f, t) = \sum_{j_1}^{n_1} \dots \sum_{j_f}^{n_f} A_{j_1 \dots j_f}(t) \prod_{\kappa=1}^m \varphi_{j_\kappa}^{(\kappa)}(Q_\kappa, t) \prod_{\kappa=m+1}^f g_{j_\kappa}^{(\kappa)}(Q_\kappa, t)$$

## ■ Variational multi-configuration Gaussian (vMCG)

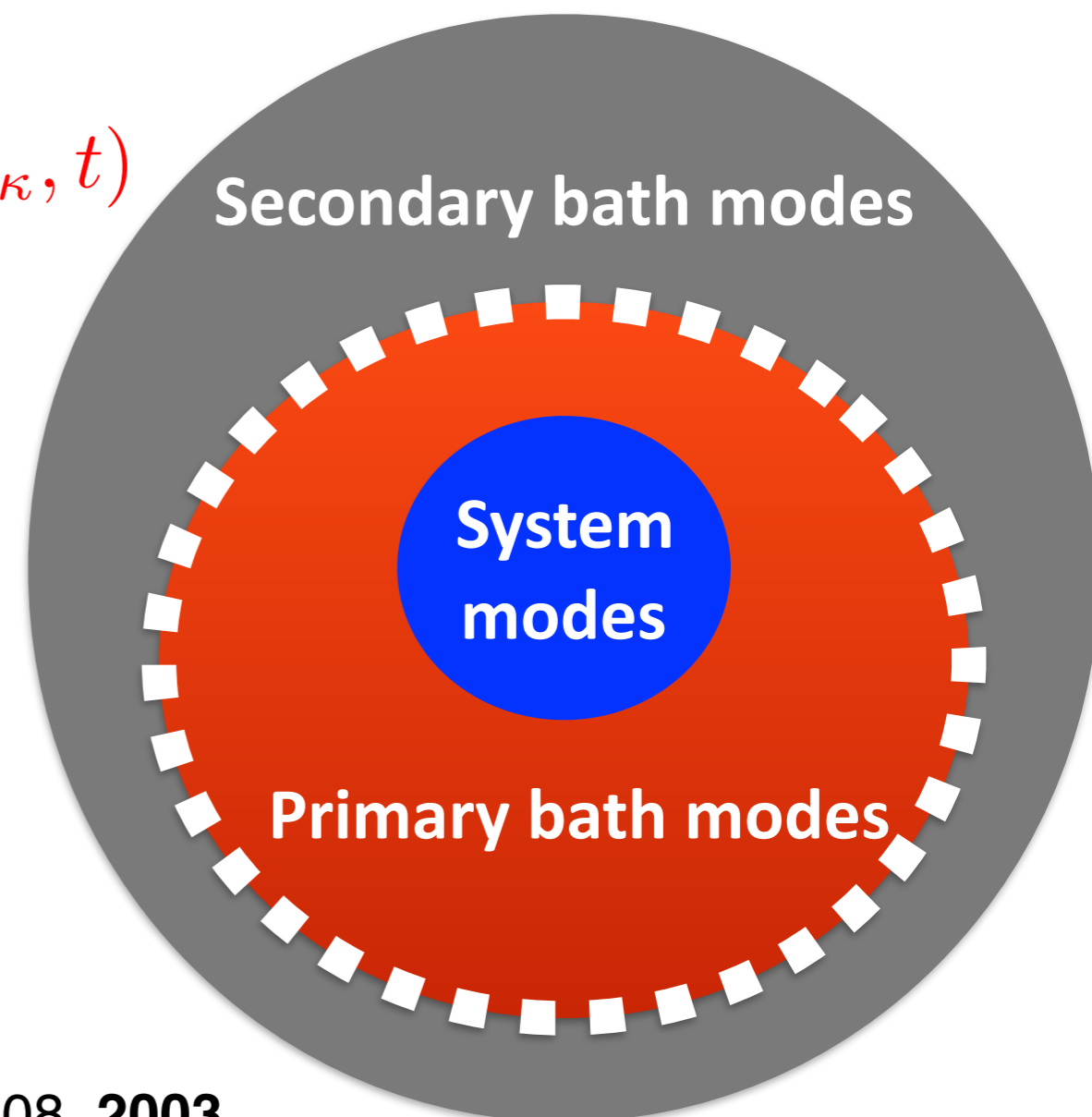
$$\Psi(Q_1, \dots, Q_f, t) = \sum_{j_1}^{n_1} \dots \sum_{j_f}^{n_f} A_{j_1 \dots j_f}(t) \prod_{\kappa=1}^f g_{j_\kappa}^{(\kappa)}(Q_\kappa, t)$$

# Gaussian based hybrid method: G-MCTDH

$$\Psi(Q, t) = \sum_J A_J(t) \Phi_J(Q, t)$$

$$\Phi_J(Q, t) = \prod_{\kappa=1}^m \varphi_{j_\kappa}^{(\kappa)}(Q_\kappa, t) \prod_{\kappa=m+1}^f g_{j_\kappa}^{(\kappa)}(Q_\kappa, t)$$

- Quantum-semiclassical MCTDH
- System-bath formalism



G. Worth, I. Burghardt, *Chem. Phys. Lett.*, 368, 502, 508, **2003**.

G. Worth, I. Burghardt, *Int. Rev. Phys. Chem.*, 34, 269, **2015**.



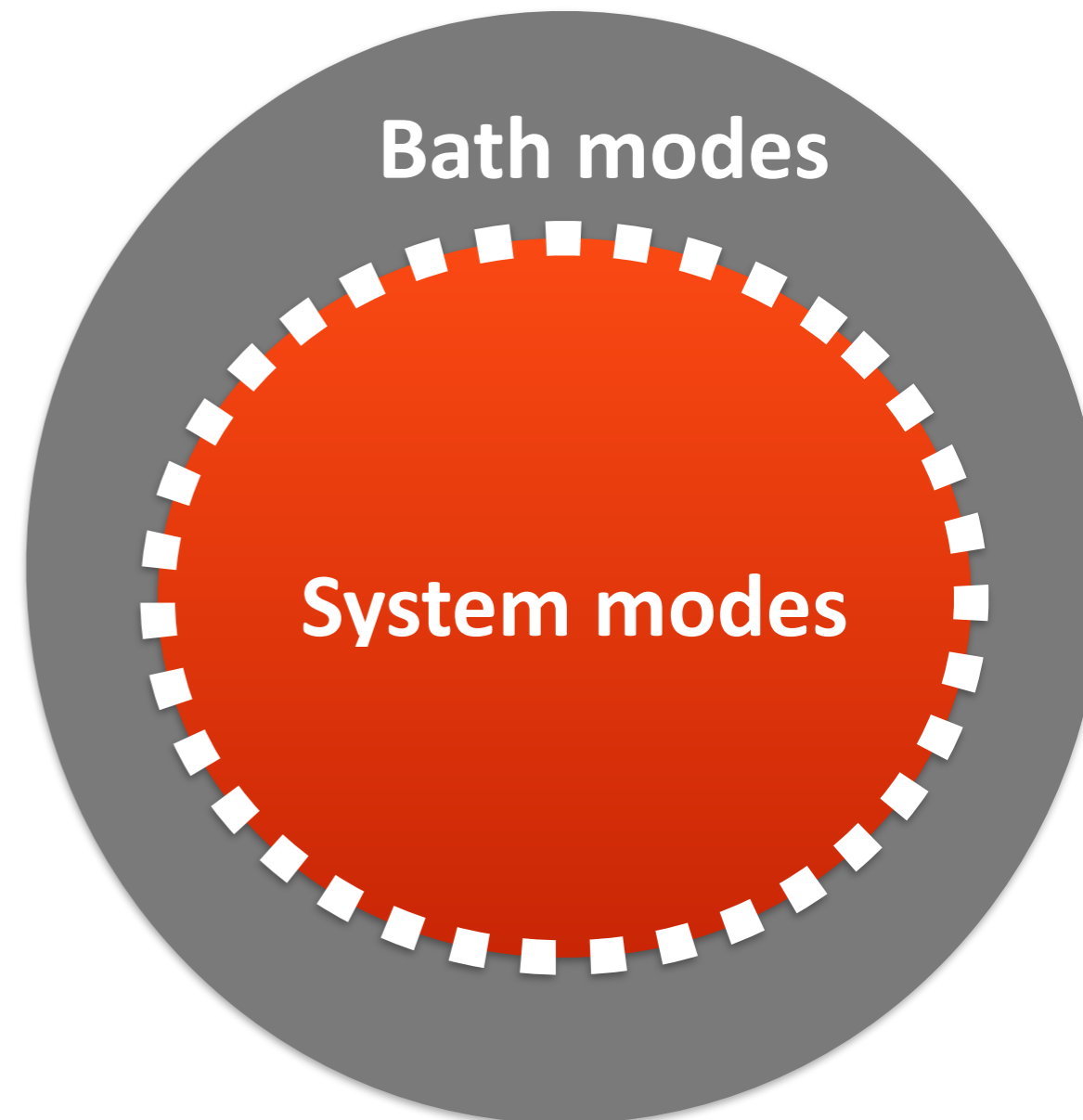
# Variational multi-configurational Gaussian (vMCG)

$$\Psi(Q, t) = \sum_J A_J(t) \Phi_J(Q, t)$$

$$\Phi_J(Q, t) = \prod_{\kappa=1}^f g_{j_\kappa}^{(\kappa)}(Q_\kappa, t)$$

$$\Psi(\mathbf{Q}, t) = \sum_{j=1}^n A_j(t) g_j(\mathbf{Q}, t)$$

- System-bath formalism
- **On-the-fly quantum dynamics**



G. Worth, I. Burghardt, *Int. Rev. Phys. Chem.*, 34, 269, **2015**.

# Type of Gaussian

$$g_{j\kappa}^{(\kappa)}(Q_\kappa, t) = \exp \left[ Q_\kappa^T \underbrace{\zeta_j^\kappa(t)}_{\text{width}} Q_\kappa + \underbrace{\xi_j^\kappa(t)}_{\text{center}} Q_\kappa + \underbrace{\eta_j^\kappa(t)}_{\text{phase}} \right]$$

$$\Lambda_j^\kappa(t) = \left( \underbrace{\zeta_j^\kappa(t)}_{\text{width}}, \underbrace{\xi_j^\kappa(t)}_{\text{center}}, \underbrace{\eta_j^\kappa(t)}_{\text{phase}} \right)$$

- Thawed Gaussian(TG)
- Separable Gaussian (SG)
- **Frozen Gaussian (FG)**
- Semi-classical motion of Gaussian center in phase-space
- Analytical integrals, localized functions, fair memory requirement

# Equation of motions for G-MCTDH

- Dirac-Frankel variational principle  $\langle \delta \Psi | H - i \frac{\delta}{\delta} | \Psi \rangle = 0$

- EOM for expansion coefficients  $i \mathbf{S} \dot{\mathbf{A}} = [\mathbf{H} - i \boldsymbol{\tau}] \mathbf{A}$

- EOM for SPFs  $i \dot{\varphi}_j^{(\kappa)} = (\hat{1} - \hat{P}^{(\kappa)}) \boldsymbol{\rho}^{(\kappa)^{-1}} \langle \hat{\mathbf{H}} \rangle^{(\kappa)} \varphi^{(\kappa)}$

- EOM for Gaussian parameters

$$i \mathbf{C}^{(\kappa)} \dot{\boldsymbol{\Lambda}}^{(\kappa)} = \mathbf{Y}$$

# Two-layer formalism

$$\Psi(\mathbf{Q}, t) = \sum_J A_J(t) \Phi_J(\mathbf{Q}, t) = \sum_J A_J(t) \prod_{\kappa=1}^f \chi_{j_\kappa}^{(\kappa)}(\mathbf{Q}_{\kappa}, t)$$

$$\chi_{j_\kappa}^{(\kappa)}(\mathbf{Q}_{\kappa}, t) = \sum_L B_{j,L}^{(\kappa)}(t) G_L^{(\kappa)}(\mathbf{Q}_{\kappa}, t) = \sum_L B_{j,L}^{(\kappa)}(t) \prod_{\mu=1}^{f^{(\kappa)}} g_{l\mu}^{\kappa,\mu}(\mathbf{Q}_{\kappa,\mu}, t)$$

# Multi-layer formalism

## ■ M-layer formalism

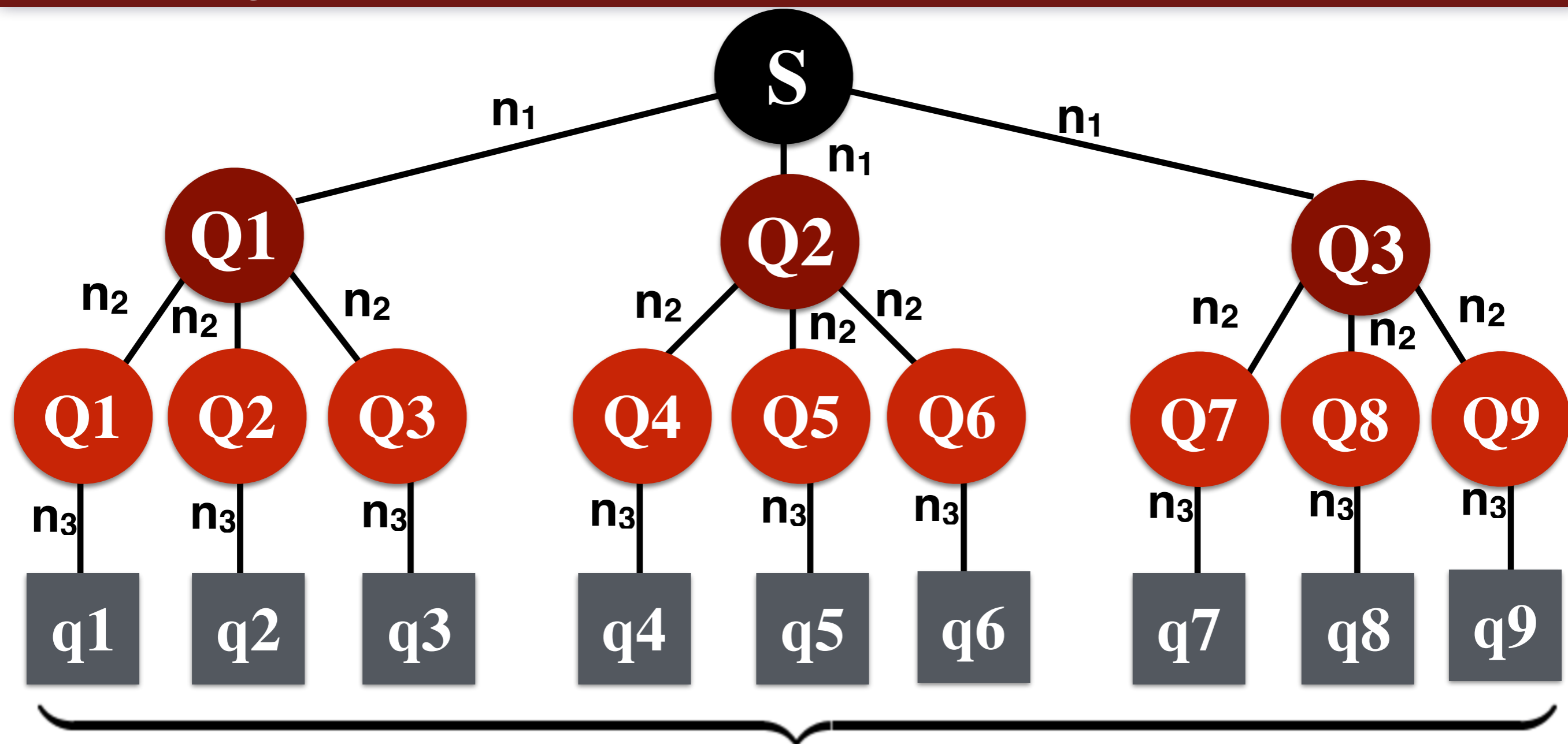
$$\Psi(\mathbf{Q}, t) = \sum_J A_J^{[1]}(t) \Phi_J^{[1]}(\mathbf{Q}, t) = \sum_J A_J^{[1]}(t) \prod_{\kappa_1=1}^{f^{[1]}} \chi_{j_{\kappa_1}}^{[1](\kappa_1)}(\mathbf{Q}_{\kappa_1}, t)$$

$$\begin{aligned} \chi_j^{[m-1](\mu_{m-1})}(\mathbf{Q}_{\kappa(m-1)}, t) &= \sum_L A_{j,L}^{[m](\mu_{m-1})}(t) \Phi_L^{[m](\mu_{m-1})}(\mathbf{Q}_{\kappa(m-1)}, t) \\ &= \sum_L A_{j,L}^{[m](\mu_{m-1})}(t) \prod_{\kappa_m=1}^{f_{\mu_{m-1}}^{[m]}} \chi_{l_{\kappa_m}}^{[m](\mu_m)}(\mathbf{Q}_{\kappa_m, \mu_{m-1}}, t) \end{aligned}$$

## ■ Last (Mth) layer is represented as FGs

$$\chi_{l_{\kappa_M}}^{[M](\mu_M)}(t) = g_{l_{\kappa_M}}^{(\mu_M)}(\Lambda_{l_{\kappa_M}}^{(\mu_M)}(t))$$

# Multi-layer vMCG is a MUST !



## Frozen Gaussian

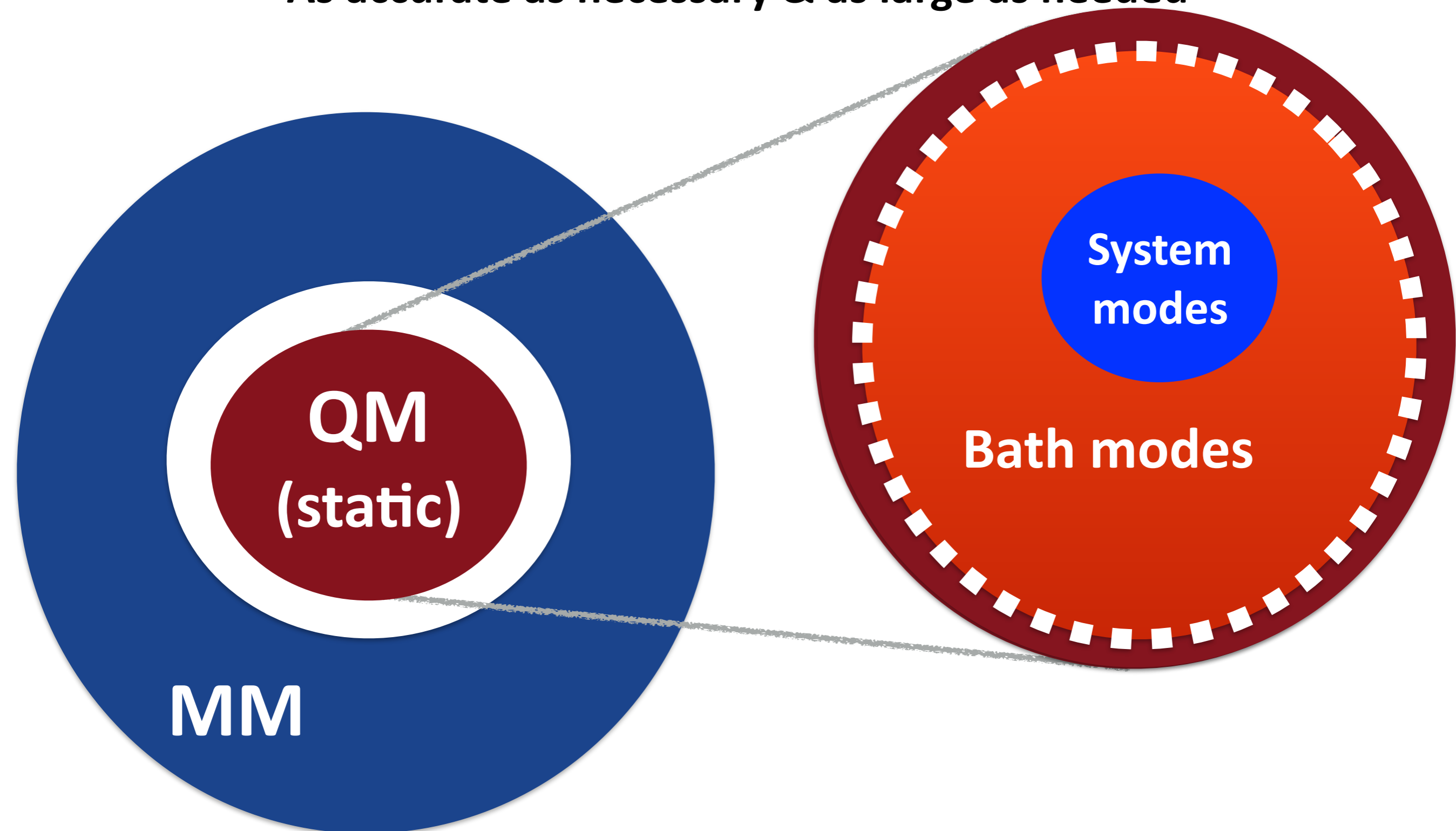
T. J. Martinez et al, *J. Phys. Chem. A*.104, 5161, 2000.

G. Worth, I. Burghardt, *Chem. Phys. Lett.*, 368, 502, 508, 2003.

G. Worth, I. Burghardt, *Int. Rev. Phys. Chem.*, 34, 269, 2015.

# Multi-scaling quantum dynamics

As accurate as necessary & as large as needed

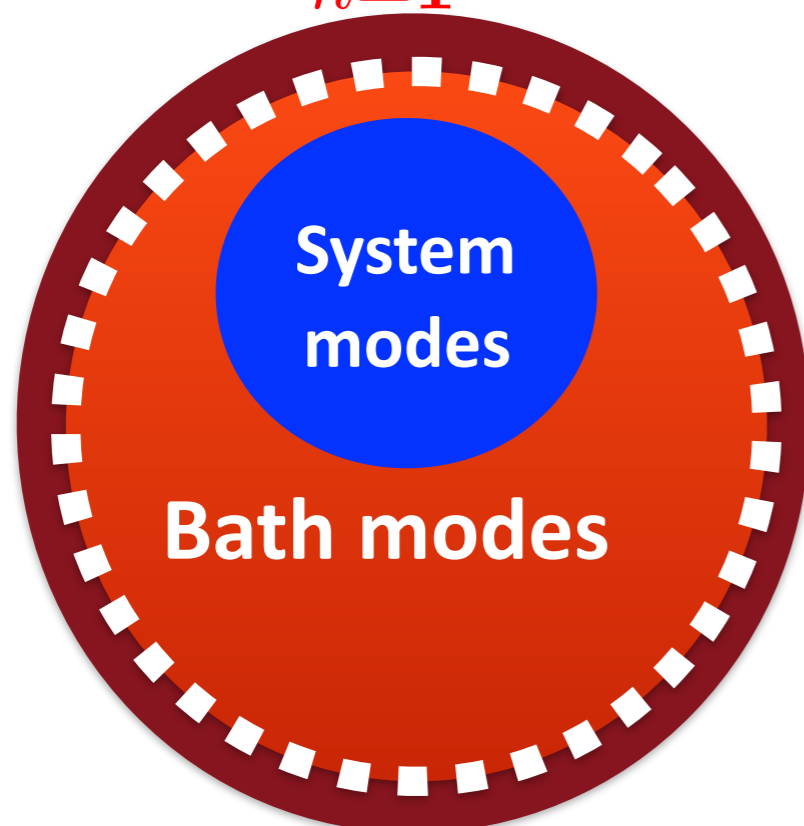


# On-the-fly-quantum dynamics

$$\Psi(\mathbf{Q}, \mathbf{t}) = \sum_{\mathbf{J}} \mathbf{A}_{\mathbf{J}}(\mathbf{t}) \Phi_{\mathbf{J}}(\mathbf{Q}, \mathbf{t})$$

$$\Phi_{\mathbf{J}}(\mathbf{Q}, \mathbf{t}) = \prod_{\kappa=1}^{\mathbf{m}} \varphi_{\mathbf{j}_{\kappa}}^{(\kappa)}(\mathbf{Q}_{\kappa}, \mathbf{t}) \prod_{\kappa=\mathbf{m}+1}^{\mathbf{f}} \mathbf{g}_{\mathbf{j}_{\kappa}}^{(\kappa)}(\mathbf{Q}_{\kappa}, \mathbf{t})$$

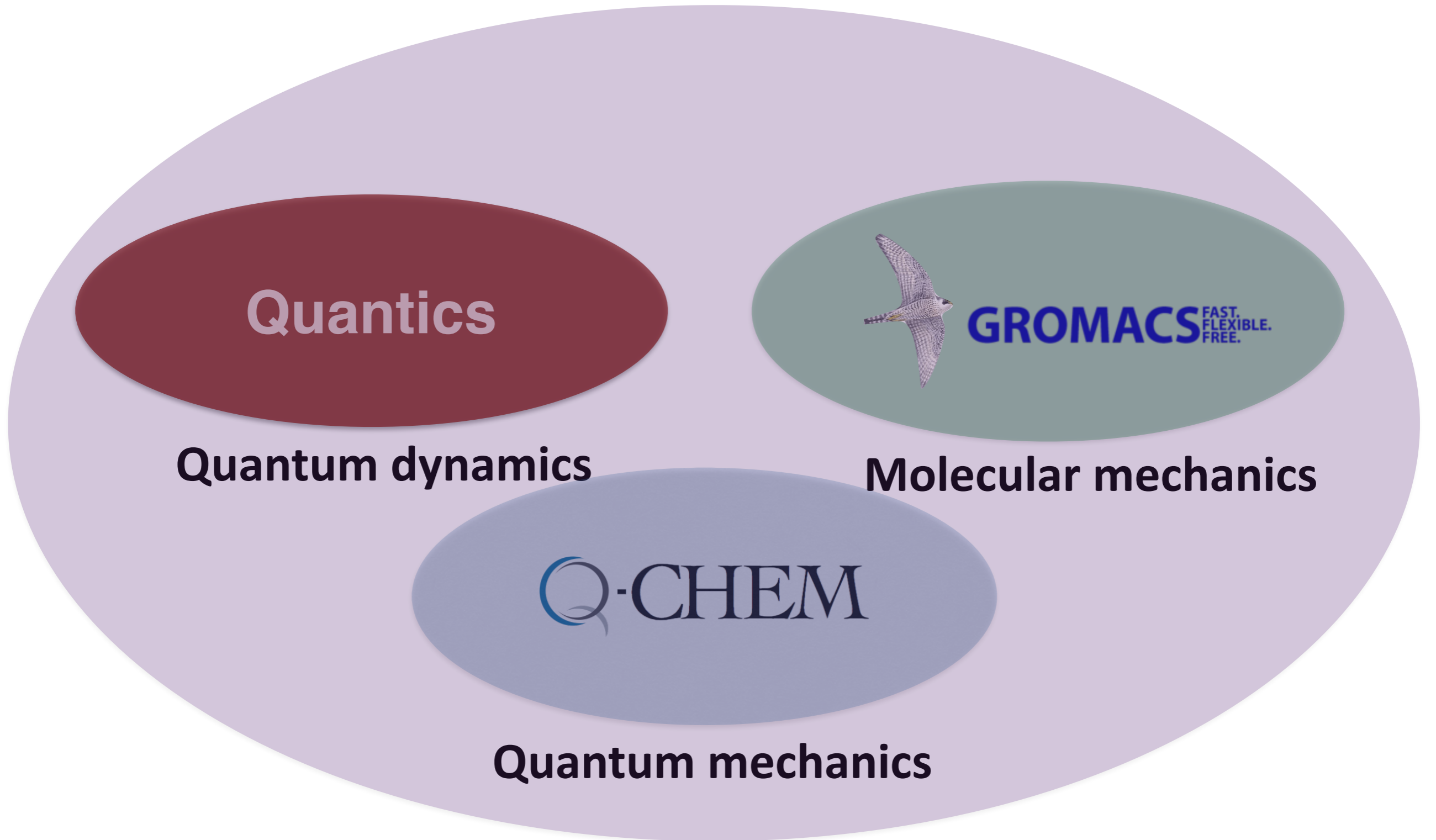
$$\Phi_{\mathbf{J}}(\mathbf{Q}, \mathbf{t}) = \prod_{\kappa=1}^{\mathbf{f}} \mathbf{g}_{\mathbf{j}_{\kappa}}^{(\kappa)}(\mathbf{Q}_{\kappa}, \mathbf{t})$$







# Software infrastructure





# On-going projects

Pr\*

I\*

Pr

Twisting

ESPT

H-bond rupture

C=C isomerization

Fluorescence

Radiationless decay

P<sub>fr</sub>

Red light

Sivasudhan Rathnachalam

Manoy Trip

Biomarkers for labeling/tracking cancer cells

State 0

State 1

State 2

hv

$S_1S_0 \pm S_0S_1$

$^1(T_1T_1)$

$T_1 + T_1$

$r_1$

$r_2$

Luis Enrique Aguilar Suarez

Singlet Fission in molecular solids

5'

3'

G-quartet 1

G-quartet 2

G-quartet 3

DG15

DG4

DG22

DG8

DA16

DG20

DG17

DA19

Kiana Moghaddam

DNA G-quadruplex and cancer therapy

G-quartet 1

G-quartet 2

G-quartet 3

G-quartet 4

Wouter Sipma

Goran Giudetti

Photo-switchable DNA in nano technology